The Anatomy of Aggregate Productivity*

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September, 2025

Abstract

We present an aggregation result that structurally dissects the drivers of aggregate productivity, i.e., technology and the reallocation of resources, across arbitrary parts of the economy using sufficient statistics that can be measured with standard datasets. Besides the typical statistics of factor shares and distortion changes, consumption share changes emerge as a new sufficient statistics that capture an income redistribution channel between households. This channel reflects how changes in households' income propagate upstream, influencing the allocation of resources across firms. We apply our results to revisit Chile's aggregate productivity stagnation since 2010, leveraging two decades of administrative firm-to-firm data. This stagnation is almost entirely driven by the reallocation of resources and in particular by expenditure changes of specific groups of the economy. Exports of mining, domestic output of manufacturing and retail, and incumbent large firms shape the bulk of this reallocation.

*We thank Hugo Hopenhayn, David Baqaee, Ariel Burstein, Michael Rubens, and John Asker for excellent guidance. Andy Atkeson, Pablo Fajgelbaum, Jonathan Vogel, Oleg Itskhoki, Sofía Bauducco, Juan Guerra provided valuable comments. The views expressed are those of the author and do not necessarily reflect the views of the Central Bank of Chile or its board members. This study was developed within the scope of the research agenda conducted by the Central Bank of Chile (CBC) in economic and financial affairs of its competence. The CBC has access to anonymized information from various public and private entities, by virtue of collaboration agreements signed with these institutions. To secure the privacy of workers and firms, the CBC mandates that the development, extraction and publication of the results should not allow the identification, directly or indirectly, of natural or legal persons. Officials of the Central Bank of Chile processed the disaggregated data. All the analysis was implemented by the authors and did not involve nor compromise the Chilean IRS. The information contained in the databases of the Chilean IRS is of a tax nature originating in self-declarations of taxpayers presented to the Service; therefore, the veracity of the data is not the responsibility of the Service.

1 Introduction

Aggregate productivity is the fundamental driver of economic growth. Despite the ubiquitous presence of aggregate productivity growth in policy debates and economic models, it continues to be a black box. In the presence of distortions, aggregate productivity evolves in response to technological progress and the reallocation of resources across agents. But which parts of the economy are the main drivers of technology and reallocation? This is a typical question when governments implement growth accounting to understand the drivers of aggregate economic growth and in policy debates, such as those involving industrial policy, where policymakers target the promotion of specific parts of the economy.

We present an aggregation result that structurally dissects the evolution of technology and the reallocation of resources across arbitrary parts of the economy using sufficient statistics that can be measured with standard datasets. In an economy with arbitrary distortions, constant-returns-to-scale technology and preferences, we show that the shocks that impact each part of the economy can be captured by the sufficient statistics of changes in factor shares, distortions and consumption shares.

These sufficient statistics measure two channels. For instance, when a distortion increases over time, it increases prices, which propagate downstream throughout the economy until it reaches final consumers, increasing their cost of living. We call this the cost-of-living channel. The sufficient statistics for this channel are the standard factor shares and distortion changes. On the other hand, the increase in a distortion can affect household demand through substitution due to changes in relative prices or through income, for instance, due to changes in income from profit ownership. These demand changes propagate upstream throughout the economy, changing the overall size of each part of the economy. We call this mechanism the income channel. The sufficient statistics for this channel are changes in households' final consumption shares.

These sufficient statistics of each part of the economy are then aggregated through a new decomposition of Domar weights. In an economy without distortions, the sales share of GDP, i.e., the Domar weight, is the appropriate measure of how to aggregate the behavior of agents. In an economy with distortions, the relevant measure is the cost-based Domar weight. Our result further decomposes the cost-based Domar weight into an arbitrary partition. Intuitively, a part of the economy, say large exporting firms from manufacturing in the northernmost part of a country, is influential if cost shocks to that part are more impactful for final consumption. Thus, besides providing a structural interpretation of how to unpack technology and reallocation, our main results allows us to structurally understand what source of variation drives cost-based Domar weights.

Beyond allowing for an arbitrary partition of the economy, our result is flexible in accommodating distortions to any input such as capital, labor and materials, arbitrary technologies and preferences as long as they are constant returns to scale, and international trade.

We apply our main result to growth accounting and revisit Chile's aggregate productivity stagnation since 2010. We leverage nearly two decades of administrative tax data of all formal firms in Chile between 2005 and 2022, including firm-to-firm linkages, which allows us to measure carefully how shocks in the economy propagate through firms' supply chains until they reach households. We partition the economy into groups of interacting sectors, locations, exporting status, and firm size. We also allow for time-varying groups of the economy, such as with firm size. For instance, a group of the economy can be firms that were small at the beginning and big at the end. We can show much this group contributed to aggregate productivity growth. Following this same logic, we allow for firm entry and exit. Therefore, we show that our disaggregation can accommodate measuring the relevance of firm dynamics.

Three objects are required to implement the structural decomposition: distortions, cost-based Domar weights, and standard aggregate objects such as aggregate factor shares and consumption shares. We focus here on explaining the first two. We measure distortions as the wedge between prices and the social marginal value of resources. This implies that wedges are the ratio between the output elasticity of an input and the input's share of revenues. We use a standard strategy from the IO literature for estimating output elasticities, albeit controlling for firm-level output and input prices. This allows us to infer quantity-based instead of revenue-based output elasticities and avoid a common critique from the literature (Bond et al., 2021). Finally, following Atkin et al. (2025), we are flexible in accommodating wedges in all inputs of production, that is, in labor, capital, and materials.

The second object we need to measure is cost-based Domar weights. These weights measure how important is firms' output for final consumption both directly and indirectly through the entire production network. This is the theory-consistent measure for aggregating firm-level changes in technology or distortions. We make progress on two fronts of how to measure this object. First, we disaggregate these weights to the aforementioned arbitrary partition. Second, we leverage firm-to-firm value-added tax data to measure all direct and indirect relationships between firms throughout the production network. The literature typically measures this object using industry input-output tables.

We find that the evolution of aggregate productivity is driven almost entirely by the reallocation of resources. Aggregate productivity in Chile increased between 2005 and 2009 but stagnated thereafter. The level of aggregate productivity in 2022 is actually lower than the one in 2009.

To dissect how reallocation of resources contributes to aggregate productivity growth, we apply our main theoretical result and measure which parts of the economy explain both the improvements in the allocation of resources during the first years and the later stagnation. The rise of allocation of resources until 2009 is driven mostly by mining exports, the domestic activity of manufacturing and retail/wholesale and incumbent large firms. The stagnation after 2010 is driven by these same groups but specially by the domestic activity of manufacturing. Financial services compensates by substantially improving the allocation of resources. For the expansion period, we show that the cost-of-living channel is the main driver. Changes in distortions and factor shares imply that resources reallocated to more efficient groups of the economy, thereby increasing efficiency and reducing overall prices faced by consumers. For the stagnation period, the income channel dominates when explaining differences across groups. Resources reallocated away from those groups of the economy, thereby reducing their demand, such as manufacturing, which in turn propagated upstream throughout the economy. Firm entry and exit explains little of the evolution of aggregate productivity.

International trade plays an important role, with varying importance across sectors. For mining, the initial improvement in allocations is mostly driven by export activity, whereas for retail it is mostly driven by domestic activity. Note that size of the sector is a poor statistic of these result as other sectors with similar value-added share such as business services matter little for the evolution of reallocation. This underscores the relevance of disaggregating the economy with the appropriate structural framework.

Regions like Santiago matter for the expansion period and other manufacturing regions matter for the stagnation period. Firms that are continuously exporting and firms that stay large throughout the period are also important groups behind the worsening of allocations.

We compare our results to three benchmarks from the literature. Each of these benchmarks ignore key dimensions of novelty in our analysis. First, we implement our growth accounting without international trade. The overall result that reallocation accounts for most of productivity growth still holds. Mining matters now little for both the expansion and stagnation periods, and retail does not contribute to the stagnation period.

Second, we implement a more standard measure of distortions by following De Loecker and Warzynski (2012), which focuses on output distortions as identified by the material wedge, and ignores the labor and capital wedge. In this version of the model and relative to the baseline model, retail/wholesale contribute much more to the stagnation through the reallocation of resources.

Finally, we implement a version aggregated at the industry level, thereby ignoring the rich granular firm and firm-to-firm data we have. In this version, exporting activity of the mining sector matters significantly more for the expansion period, whereas the domestic side of the financial sector contributes almost twice to reallocation in the stagnation period. These counterfactual models highlight that each of these ingredients are important for appropriately disaggregating aggregate productivity and identifying which groups are relevant.

Related Literature. We relate to three strands of the literature. First, to the literature on aggregation in macroeconomics. With new theory developments and systematic access to more and new administrative micro datasets, significant progress has been made in understanding how to aggregate starting from the behavior of agents. We build on Baqaee and Farhi (2019) who present a strategy for aggregation in the context of distorted economies in open economies which, in turn, is the modern version of the work by François Quesnay in his Tableau Economique (1758). We extend Baqaee and Farhi (2019) to structurally identify which parts of the economy drive technology and reallocation, which has been a black box so far. A new channel and sufficient statistics emerge when disaggregating reallocation. The new channel captures the role that redistribution of income and changes in demand play for the allocation of resources. This channel is captured by the sufficient statistics of changes in consumption shares, which can be measured using standard datasets. A side product of the aggregation result is that we provide a strategy on how to structurally decompose the cost-based Domar weight, a sufficient statistics that is essential for aggregation.

Second, we connect to the literature that studies how distortions impact aggregate productivity. Going back to Restuccia and Rogerson (2008), Hsieh and Klenow (2010) and Hsieh and Klenow (2009), it has been argued that distortions can be an important driver of aggregate productivity differences between countries. This argument has been extended recently by Baqaee and Farhi (2020) to a more general framework with flexible production and preferences in general equilibrium. We extend this framework by further structurally decomposing technology and reallocation into an arbitrary partition of the economy.

The previous literature has had an important impact on growth accounting in terms of studying what shapes GDP growth and how it accounts for income differences across countries. The seminal paper of Hsieh and Klenow (2009) offered a new perspective of how distortions can play a role in accounting for differences in aggregate productivity but in partial equilibrium. Baqaee and Farhi (2019) applied their insights into growth accounting and found that allocative efficiency plays a big role in shaping aggregate productivity growth in general equilibrium. Here we contribute by further pushing on disaggregating technology and the reallocation of resources into arbitrary parts of the economy using standard datasets, and thus advancing the analysis of structurally microfounding aggregate productivity growth.

Finally, we relate to the literature of how international trade shapes the allocation of resources and aggregate productivity. The closest papers are Burstein and Cravino (2015), Blaum et al. (2018) and Kehoe and Ruhl (2008).[1] To this literature we contribute by presenting a formula for growth accounting in open economies with distortions that is general, feasible to arbitrary disaggregations, not subject to parametric assumptions other than constant returns to scale in preferences and technology, and implementable with standard datasets. A common result in the literature is that international trade contributes to aggregate productivity through resource reallocation between firms. We show that this logic is generalized to a world with distortions and that the reallocation of resources is the main driver of aggregate productivity. Also, relative to this literature, we highlight not only the role of imported intermediate inputs but also the role of exports in driving

[1] We also connect to the literature that highlights the impact of trade indirectly through production networks (Dhyne et al., 2023). This literature mostly focuses on settings without distortions. We show how this mechanism interacts with distortions, and how both affect aggregate TFP growth.

improvements in aggregate TFP and how to disaggregate them into different parts of the economy.

The remainder of the paper is organized as follows. Section 2 presents the theory. Section 3 describes the data. Section 4 presents the measurement strategy. Section 5 describes the results and Section 6 concludes.

2 Theory

This section presents a theory of a general equilibrium economy with arbitrary distortions. We follow the presentation from Baqaee and Farhi (2019) and Atkin et al. (2025). We start with a closed economy and then extend the model to an open economy.

The economy consists of G groups of households, F primary factors, and N firms. Each firm has access to its own arbitrary, but constant returns-to-scale (CRS), production function that uses potentially all factors and other goods as inputs.[2]

Total output of a firm i is

$$q_i = A_i F_i \left(\left\{ q_{ij} \right\}_{j \in \mathcal{N}}, \left\{ \bar{L}_{if} \right\}_{f \in \mathcal{F}} \right),$$

where A_i is total factor productivity, q_{ij} are the intermediate goods input from other $j \in \mathcal{N}$ firms, and $\bar{L}_{if}$ are the firms' use of primary production factors.

The economy is subject to distortions on both output and inputs. As in Atkin et al. (2025), we allow for a flexible treatment of input distortions. When a firm i buys an input (a good or a factor) from a supplier j at price p_j^S,it behaves as-if the marginal cost to firm i is $\tau_{ij} p_j^S$ rather than the price p_j^S that the supplier receives. When this is the case, the supplier's and buyer's marginal incentives are not aligned, which results in an input distortion (or wedge). The source of such distortions could be many. One example is labor regulation, where the government can mandate firms to spend more resources on labor, for instance, on safety purposes. These are not payments that workers receive, but rather internal costs that the firm spends resources on when hiring another worker. Regardless of their underlying source, the combined effect of input distortions determines the value of τ_{ij}, which we treat as a primitive.

In addition to arbitrary input distortions τ_{ij}, we also allow for output distortions, μ_i. This distortion drives a wedge between the price p_i that the customer of firm i pays and the marginal cost of producing by firm i, c_i. Thus, firms minimize cost given input prices and charge the output price $p_i = \mu_i c_i$.

We partition households into G groups according to their final demand. These groups can be set at an arbitrary level. For instance, a group $g \in \mathcal{G}$ can be products produced by industry-location pairs such as manufacturing-north, where g represents the group of households that demand

[2]Given the CRS assumption, separating the production function into more disaggregated production units is without loss of generality. Thus, we can accommodate dividing the firm into multiple fictitious firms. These fictitious firms can, for instance, produce different products or output for different buyers.

manufacturing goods from the northernmost location.[3]

Each group of households, indexed by g, has access to its own arbitrary, but homothetic, utility that consumes potentially all goods. Utility of a group g is

$$U_g = D_g(y_{g1}, \dots, y_{gN}) \tag{1}$$

where y_{gi} is group g's final demand of good i. Each group faces the budget constraint

$$\sum_{i \in \mathcal{N}} p_i y_{gi} = \sum_{f \in \mathcal{F}} w_f L_{gf} + \sum_{i \in \mathcal{N}} \pi_{gi} \tag{2}$$

where p_i and y_{gi} are the price and quantity of good i consumed by group g, w_f and L_{gf} are the price and quantity of the factor f owned by group g, π_{gi} is profits of firm i belonging to group g. Finally, GDP is the numeraire.

Market clearing conditions are the following. For goods $i \in \mathcal{N}$,

$$q_i = \sum_{g \in \mathcal{G}} y_{gi} + \sum_{j \in \mathcal{N}} q_{ji}.$$

For factors, we have the following market clearing conditions:

$$L_f = \sum_{i \in \mathcal{N}} \bar{L}_{if} = \sum_{g \in \mathcal{G}} L_{gf},$$

for all $f \in \mathcal{F}$. L_f is total factor supply of f, which we assume is exogenous.

General Equilibrium

Given productivity A_i, output distortions μ_i, input distortions τ_{ij}, the general equilibrium is the set of prices p_i, intermediate input choices q_{ij}, factor input choices $\{\bar{L}_{if}\}$, output q_i, and consumption choices y_{gi}, such that: (i) each producer minimizes its costs and charges a price of each good that is equal to its output distortion multiplied by its marginal cost; (ii) each household group maximizes utility subject to its budget constraint taking prices as given; and (iii) markets clear for all goods and factors.

[3]Given that households can consume goods across different supplier firms, we split final consumption into the sets $\mathcal{G}$. Naturally, these sets can also represent household demographic groups. As we will show though, that partitioning requires another type of data.

National Accounts

Define final nominal expenditure of group g of households by

$$E_g \equiv \sum_{i \in \mathcal{N}} p_i y_{gi},$$

and final demand of good i by $y_i = \sum_{g \in \mathcal{G}} y_{gi}$.

Using the expenditure approach, nominal GDP is the sum of final demand,

$$GDP = \sum_{i \in \mathcal{N}} p_i y_i = \sum_{g \in \mathcal{G}} E_g.$$

Consumption shares for each final output are described by the following vector b_i:

$$b_i = \begin{cases} \frac{p_i y_i}{GDP} & \text{if } i \in \mathcal{N} \\ 0 & \text{otherwise} \end{cases}$$

and consumption shares for each final output and household group are described by

$$b_{gi} \equiv \frac{p_i \, y_{gi}}{\text{GDP}},$$

so that $b_i = \sum_{g \in \mathcal{G}} b_{gi}$.

Given the definition of GDP, the GDP deflator is defined as a chained index as follows:

$$d \log P = \sum_{i \in \mathcal{N}} \frac{p_i y_i}{GDP} d \log p_i,$$

which can be expressed in vector form using the consumption share b_i as,

$$d \log P = b' d \log p,$$

where $d \log p$ is a vector of $N + F$ prices. Then, real GDP growth is computed by chaining absolute indices:

$$d \log Y = d \log GDP - d \log P.$$

For each group g, we define its real index[4] by

$$d \log Y_g = d \log E_g - \sum_{i \in \mathcal{N}} \frac{p_i y_{gi}}{E_g} d \log p_i,$$

[4] This real index provides a first-order approximation of the equivalent variation around the initial equilibrium. For a more detailed discussion, see Baqaee and Burstein (2023).

$$= d\log E_g - \frac{GDP}{E_g}\sum_{i\in\mathcal{N}} b_{gi} d\log p_i.$$

Thus, to a first order,

$$\begin{aligned} d\log Y &= d\log GDP - d\log P \\ &= \sum_{g\in\mathcal{G}} \frac{E_g}{GDP} d\log E_g - \sum_{g\in\mathcal{G}}\sum_{i\in\mathcal{N}} \frac{p_i y_{gi}}{GDP} d\log p_i \\ &= \sum_{g\in\mathcal{G}} B_g \left(d\log E_g - \sum_{i\in\mathcal{N}} \frac{p_i y_{gi}}{E_g} d\log p_i \right) \\ &= \sum_{g\in\mathcal{G}} B_g d\log Y_g \end{aligned}$$

Thus, the change in *real* GDP can be expressed, to a first order, as a sum of each subgroup's real-index change, weighted by $B_g = \frac{E_g}{GDP}$.

Finally, we define aggregate factor shares as

$$\Lambda_f \equiv \frac{w_f L_f}{GDP},$$

for all $f \in \mathcal{F}$.

Input-Output Objects

The input-output (IO) matrix groups all firm-to-firm transactions and factor expenditures in a matrix of dimensions $(N + F) \times (N + F)$. We define the revenue-based IO matrix Ω, where the ij^{th} element captures the expenditure of firm i on inputs of supplier j as a share of firm i total revenue, $p_i q_i$, where q_i is the physical production,[5]

$$\Omega_{ij} \equiv \frac{p_j^S q_{ij}}{p_i q_i}$$

The cost-based IO matrix $\tilde{\Omega}$ describes the share of expenditures in firms' total costs. By Shepherd's Lemma, the expenditure share of production costs for firm i from an origin j is

$$\tilde{\Omega}_{ij} = \frac{\text{Value of input j used by firm i}}{\text{Firm i total cost}} = \frac{\tau_{ij} p_j^S q_{ij}}{\sum_{k\in\mathcal{N},\mathcal{F}} \tau_{ik} p_k^S q_{ik}}.$$

The cost-based and the revenue-based IO matrices are related trough wedges following $\tilde{\Omega}_{ij} =$

[5]Note that a supplier j can be a firm or a factor.

$\tau_{ij}\mu_j\Omega_{ij}$. The cost-based Leontief inverse matrix $\tilde{\Psi}$ accounts for both direct and indirect cost exposures of every firm through the economy's production network. Each element of $\tilde{\Psi}$ measures the weighted sums of all paths (steps) of all length size from producer j to producer i.

$$\tilde{\Psi} \equiv (I - \tilde{\Omega})^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \dots$$

We define cost-based Domar weights as $\tilde{\lambda}$ for firms and $\tilde{\Lambda}$ for factors, as the interaction of firms and factors GDP exposure (measured by b) with their relevance throughout the production network (measured by $\tilde{\Psi}$),

$$\tilde{\lambda}' \equiv b'\tilde{\Psi}.$$

Given the partition of final demand into G groups each indexed by g, we can define the disaggregated cost-based Domar weights as:

$$\tilde{\lambda}_{gi} \equiv \sum_{k \in \mathcal{N}} b_{gk}\tilde{\Psi}_{ki} \quad \text{for } i \in \mathcal{N}$$

$$\tilde{\Lambda}_{gf} \equiv \sum_{i \in \mathcal{N}} b_{gk}\tilde{\Psi}_{kf} \quad \text{for } f \in \mathcal{F}$$

By construction, we get:

$$\tilde{\lambda}_i = \sum_{g \in \mathcal{G}} \tilde{\lambda}_{gi},$$

$$\tilde{\Lambda}_f = \sum_{g \in \mathcal{G}} \tilde{\Lambda}_{gf}.$$

In other words, this represents a linear decomposition of the Domar weights into groups of households according to their final demand. We provide a structural interpretation of the cost-based Domar weights and this linear decomposition in the following section.

2.1 Ex-Post Sufficient Statistics

In this section we derive comparative-static results in terms of ex-post reduced-form sufficient statistics that extend Baqaee and Farhi (2020) to analyze the origins of aggregate output across groups of households in the economy. Take an allocation matrix $\mathcal{X}$ which captures admissible allocation of resources, where each of its elements $\mathcal{X}_{ij} = q_{ij}/y_j$ is firm j output share used in production by firm i. All feasible allocations are defined by an allocation matrix $\mathcal{X}$, a vector of productivity A, a vector of distortions $\tau\mu$,[6] and a vector of factor supplies, F. In particular, the

[6] For the purpose of the comparative statics we will implement, it is not necessary to separate out the role of τ versus μ. This is without loss and can be easily implemented if one can measure τ separately from μ.

equilibrium allocation yields an allocation matrix $\mathcal{X}(A, F, \tau\mu)$, which in turn generates an output level of $\mathcal{Y}(A, \mathcal{X}(A, F, \tau\mu))$.

The effect on real GDP of each group g from a productivity shock ($\mathrm{d}\log A$) and a distortion shock ($\mathrm{d}\log\tau\mu$) can be decomposed into a pure change in technology ($\mathrm{d}\log A$) for a given fixed allocation matrix $\mathcal{X}$ and the change in the distribution of resources allocation matrix ($\mathrm{d}\mathcal{X}$) holding technology constant. The following proposition formally decomposes these effects at the group level.

Proposition 1 (Decomposition of Output Changes from Technology and Reallocation at the Group Level)**.**

$$d\log\mathcal{Y} = \sum_{g\in G} \underbrace{\frac{\partial\log\mathcal{Y}}{\partial\log\mathcal{Y}_g}\frac{\partial\log\mathcal{Y}_g}{\partial\log A}d\log A}_{\Delta\ \textit{Technology of Group } g} + \sum_{g\in G} \underbrace{\frac{\partial\log\mathcal{Y}}{\partial\log\mathcal{Y}_g}\frac{\partial\log\mathcal{Y}_g}{\partial\mathcal{X}}d\mathcal{X}}_{\Delta\ \textit{Reallocation of Group } g} \tag{3}$$

where $d\mathcal{X} = \frac{d\mathcal{X}}{d\log A}d\log A + \frac{d\mathcal{X}}{d\log\tau\mu}d\log\tau\mu$*. The proof is obtained by applying the chain rule to the aggregate decomposition from Baqaee and Farhi (2020):* $d\log\mathcal{Y} = \frac{\partial\log\mathcal{Y}}{\partial\log A}d\log A + \frac{\partial\log\mathcal{Y}}{\partial\mathcal{X}}d\mathcal{X}$*.*

This proposition decomposes the aggregate output changes from technology ($d\log A$) and reallocation ($d\mathcal{X}$) into group-level contributions. It decomposes the technological effect (first component) that holds fixed the allocation of resources, and the reallocation effect (second component) that comes from the reallocation of resources into the different groups of the economy that drive those changes. The total effect is the sum of each group's impact, where a group's contribution is determined by how technology and reallocation affect its output ($\frac{\partial\log\mathcal{Y}^g}{\partial\log A}\mathrm{d}\log A$ and $\frac{\partial\log\mathcal{Y}^g}{\partial\mathcal{X}}\mathrm{d}\mathcal{X}$), weighted by that group's importance to aggregate production ($\frac{\partial\log\mathcal{Y}}{\partial\log\mathcal{Y}^g}$).

Note that, compared to Baqaee and Farhi (2020), we structurally decompose technology and reallocation into arbitrary groups of the economy to understand which parts of the economy undergo technological change and reallocation of resources. This decomposition operates through the real indices of each subgroup g, Y_g, which, as defined above, are changes in nominal expenditure adjusted for price changes.[7]

Change in real GDP can be expressed as a weighted sum of changes in these group-specific real indices, where the weights represent each subgroup's share in GDP. This approach allows us to trace how productivity and reallocation changes propagate through the economy for specific groups and ultimately contribute to aggregate GDP growth. By examining the real indices of different groups, we can identify which parts of the economy benefit most from technological improvements and resource reallocation.

While Proposition 1 provides the conceptual framework, applying it to data requires expressing the technology and reallocation terms using observables. Proposition 2 characterizes the

[7]Note that, compared to Baqaee and Farhi (2020), we call the non-technology term reallocation instead of allocative efficiency. This is because, when decomposing this term across groups of households, some ingredients will represent changes in the allocation of resources that are not related to efficiency.

change in output in response to changes in productivity and distortions in inefficient economies for an arbitrary partition of the economy. The formula below expresses this decomposition in terms of measurable sufficient statistics.

Proposition 2. *Consider a distribution of resources $\mathcal{X}$ corresponding to the general equilibrium allocation at the point $(A, \mu\tau)$ and an arbitrary partition of final demand into G groups indexed by g, then*

$$\frac{d\log Y}{d\log A_i} = \sum_{g\in\mathcal{G}} \tilde{\lambda}_{gi} - \sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}} \tilde{\Lambda}_{gf}\frac{d\log\Lambda_f}{d\log A_i} + \sum_{g\in\mathcal{G}} B_g \frac{d\log B_g}{d\log A_i},$$

$$\frac{d\log Y}{d\log \tau_{ij}\mu_i} = -\sum_{g\in\mathcal{G}} \tilde{\lambda}_{gi}\tilde{\Omega}_{ij} - \sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}} \tilde{\Lambda}_{fg}\frac{d\log\Lambda_f}{d\log \tau_{ij}\mu_i} + \sum_{g\in\mathcal{G}} B_g \frac{d\log B_g}{d\log \tau_{ij}\mu_i}$$

Furthermore, the decomposition of output changes into pure changes in technology and reallocation is given by

$$d\log Y = \sum_{g\in\mathcal{G}} \underbrace{\sum_{i\in\mathcal{N}} \tilde{\lambda}_{gi} d\log A_i}_{\Delta\ \text{Technology of Group } g} + \sum_{g\in\mathcal{G}} \underbrace{\left(-\sum_{i\in\mathcal{N}} \tilde{\lambda}_{gi} \sum_{j\in\mathcal{N},\mathcal{F}} \tilde{\Omega}_{ij} d\log\tau_{ij}\mu_i - \sum_{f\in\mathcal{F}} \tilde{\Lambda}_{gf} d\log\Lambda_f + dB_g \right)}_{\Delta\ \text{Reallocation of Group } g}$$

Proposition 2 provides a structural decomposition of the macroeconomic impact of microeconomic productivity and distortion shocks in terms of observables. Within this decomposition, each term inside the brackets captures how changes in technology and reallocation of group g affect real GDP, through the real index of each group.

Proposition 2 also shows the role that reallocation of resources can have. There are potentially two forces at play. When the initial equilibrium is inefficient, resources are misallocated and thus equilibrium changes in the allocation of resources, $d\mathcal{X}$, can lead to changes in output even to the first order. This is what is called a change in allocative efficiency and is given by a simple sufficient statistic. It is captured by a weighted average of the reductions in factor shares, $d\log\Lambda_f$, with weights given by cost-based factor shares, $\tilde{\Lambda}_{gf}$. A reduction in the weighted average of factor shares indicates that resources are reallocated to the more (downwardly) distorted parts of the economy that had higher share of profits (and thus lower factor share of revenues). Allocative efficiency improves because, from a social perspective, these distorted parts of the economy were too small to begin with. This result implies a structural interpretation of $\tilde{\Lambda}_f$ and $\tilde{\Lambda}_{gf}$. The cost-based Domar weight $\tilde{\Lambda}_f$ measures the direct and indirect exposure of aggregate output to the change in factor shares, $d\log\Lambda_f$, the sufficient statistics representing allocative efficiency.

The disaggregated cost-based Domar weight $\tilde{\Lambda}_{gf}$ measures the same downstream propagation but only through the subset of firms that are connected through supply chains to the final demand of group g. Specifically, the weight $\tilde{\Lambda}_{gf}$ measures group g's direct and indirect exposures on factor f through the entire supply chain. Groups more exposed to distorted factors benefit disproportionately from reallocation when factor-specific distortions decrease, thus improving their allocative efficiency. Thus, this provides a structural decomposition of which groups of the economy drive allocative efficiency changes.

A similar intuition holds for the case of distortion shocks. Proposition 2 implies that changes in allocative efficiency of each group g is given by a simple sufficient statistic, $-\sum_{i\in\mathcal{N}}\tilde{\lambda}_{gi}\sum_{j\in\mathcal{N},\mathcal{F}}\tilde{\Omega}_{ij}d\log\tau_{ij}\mu_i-\sum_{f\in\mathcal{F}}\tilde{\Lambda}_{gf}d\log\Lambda_f$. The reduction in factor shares, $\sum_{f\in\mathcal{F}}\tilde{\Lambda}_{gf}d\log\Lambda_f$, reflects both the direct reduction of factor shares from increased distortions for a given allocation of resources, $\sum_{i\in\mathcal{N}}\tilde{\lambda}_{gi}\sum_{j\in\mathcal{N},\mathcal{F}}\tilde{\Omega}_{ij}d\log\tau_{ij}\mu_i$, and the reallocation of resources towards or away from more distorted producers. To isolate the changes in allocative efficiency, one needs to net out the direct impact of distortion changes. As with cost shocks, Proposition 2 shows how to structurally decompose the impact of distortion shocks into how groups of the economy are affected through the propagation of the shock in the economy. That is, a structural decomposition of the propagation of shocks.

A second force at play are reallocation of resources that are not necessarily associated with efficiency changes, but still matter potentially for different groups of the economy. This second force is captured by dB_g, changes in group g's final expenditure shares. For instance, when productivity changes, it can affect the total nominal income of different groups of households depending on their source of income, which in turn can change the expenditure of those groups. Expenditure of households can also change due to substitution effects given changes in relative prices. All those forces affect aggregate GDP and are captured, to a first order, by the sufficient statistics dB_g. These changes in demand propagate upstream in the economy, affecting factor demand and thus the allocation of resources. This term can be driven also by changes in distortions. This term is absent in the setting of Baqaee and Farhi (2020) because, since GDP is the numeraire, one has that $\sum_{g\in\mathcal{G}}B_g=0$[8].

The factor share and distortion term described previously relate to forward propagation in the price index through consumption exposure. We call this reallocation the cost-of-living channel. The expenditure share term (dB_g) captures backward propagation through income as shocks alter upstream factor demand, leading to redistribution effects between groups of households. We call this reallocation the income channel.

With both productivity and distortion shocks, Proposition 2 shows how to structurally decompose what share of technology changes and reallocation are driven by different groups of the economy. The key objects that allow to implement such a decomposition are the disaggregated cost-based Domar weights, $\tilde{\lambda}_{gi}$, $\tilde{\Lambda}_{gf}$ and dB_g. This is our main theoretical result that allows us to dissect the anatomy of aggregate output.

2.2 Examples

We present three examples to provide more intuition of our main result. While our theoretical framework relies on sufficient statistics for ex-post analysis, based on observable changes in expenditure and factor shares, the following examples specify model primitives to trace the en-

[8]Thus, in the aggregate across groups, the reallocation term in Proposition 2 corresponds to the allocation efficiency term in Baqaee and Farhi (2020)

dogenous responses of shocks to primitives. By explicitly solving for the ex-ante problem, we illustrate how different economic structures generate distinct patterns of reallocation through the cost-of-living and income channels for different groups in the economy. We focus on changes in distortions, although one can also implement productivity changes.

Example 1: Horizontal Economy with Symmetric Preferences

Consider a horizontal economy with two final goods produced by Firm 1 and Firm 2 using labor: $Y_i = A_i L_i$. Two household groups have identical Cobb-Douglas preferences, with expenditure shares B_1 and B_2. Household 1 owns labor, while Household 2 owns firms. Firms minimize their costs and impose exogenous distortions μ_1 and μ_2, respectively.[9] This setup illustrates how symmetric preferences affect the disaggregation of aggregate output.

Define the aggregate distortion as[10]:

$$\mathcal{M} = \left(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}\right)^{-1} \tag{4}$$

When Firm 1's output distortion increases ($d\log\mu_1 > 0$), aggregate output changes according to:

$$\begin{aligned} d\log Y &= \tilde{\lambda}_1 d\log\mu_1 + d\log\Lambda_L \\ &= \tilde{\lambda}_1\left(\frac{\mathcal{M}}{\mu_1} - 1\right) d\log\mu_1 \end{aligned}$$

where $d\log\Lambda_L = -\tilde{\lambda}_1\frac{\mathcal{M}}{\mu_1} d\log\mu_1$. This corresponds to the reallocation term in Proposition 1. When $\mu_1 < \mu_2$, we have $\mathcal{M} > \mu_1$, so $d\log Y > 0$. Resources reallocate toward the initially underproducing good 2.

Following Proposition 2, the disaggregation into the contribution of both types of households is as follows:

$$d\log Y = \underbrace{\underbrace{\left[\tilde{\lambda}_{11}\left(\frac{\mathcal{M}}{\mu_1} - 1\right) d\log\mu_1\right]}_{-B_1 d\log P_1 \text{ (cost-of-living channel)}} + \underbrace{\left[-\frac{\mathcal{M}}{\mu_1}\lambda_1 d\log\mu_1\right]}_{dB_1 \text{ (income channel)}}}_{\text{Reallocation of Household 1}} + \underbrace{\underbrace{\left[\tilde{\lambda}_{21}\left(\frac{\mathcal{M}}{\mu_1} - 1\right) d\log\mu_1\right]}_{-B_2 d\log P_2 \text{ (cost-of-living channel)}} + \underbrace{\left[\frac{\mathcal{M}}{\mu_1}\lambda_1 d\log\mu_1\right]}_{dB_2 \text{ (income channel)}}}_{\text{Reallocation of Household 2}} \tag{5}$$

[9] For simplicity, we abstract from input distortions in these examples.

[10] This measure corresponds to the harmonic mean of output wedges across sectors, with weights given by the revenue-based Domar weights.

Regrouping by household:

$$d\log Y = \underbrace{B_1 d\log Y_1}_{\text{Household 1 contribution}} + \underbrace{B_2 d\log Y_2}_{\text{Household 2 contribution}} \tag{6}$$

With symmetric preferences, both households face the same price index change. The cost-of-living channel $\tilde{\lambda}_{g1}\left(\frac{\mathcal{M}}{\mu_1}-1\right)d\log\mu_1$ affects them proportionally to their expenditure shares. The income channel works through factor share changes. Increases in output distortion reduce Λ_L, transferring income from labor-owning Household 1 to firm-owning Household 2. This change in income increases the expenditure share of Household 2 and reduces the one from Household 1. Thus, whereas both households gain through the cost-of-living channel, there is unequal impacts through the income channel.

Example 2: Horizontal Economy with Heterogeneous Preferences

Now consider one difference: heterogeneous preferences. Household 1 consumes only good 2, while Household 2 consumes only good 1. This illustrates how the disaggregated Domar weights $\tilde{\lambda}_{gi}$ capture group-specific exposure to distortions. Consider the same output distortion shock as before, $d\log\mu_1 > 0$.

Proposition 2 implies:

$$\begin{aligned} d\log Y = & \underbrace{[0]}_{-B_1 d\log P_1 \text{ (cost-of-living channel)}} + \underbrace{\left[-\frac{\mathcal{M}}{\mu_1}\lambda_1 d\log\mu_1\right]}_{dB_1 \text{ (income channel)}} \\ & + \underbrace{\left[-B_2\left\{\lambda_1\frac{\mathcal{M}}{\mu_1}-1\right\}d\log\mu_1\right]}_{-B_2 d\log P_2 \text{ (cost-of-living channel)}} + \underbrace{\left[\frac{\mathcal{M}}{\mu_1}\lambda_1 d\log\mu_1\right]}_{dB_2 \text{ (income channel)}} \end{aligned} \tag{7}$$

The heterogeneous consumption baskets create asymmetric forward propagation in the cost-of-living channel. Since Household 1 does not consume good 1, we get $\tilde{\lambda}_{11} = 0$. Household 2 faces full exposure since it is the only group consuming the good that is shocked, $\tilde{\lambda}_{21} = B_2$.[11]

Example 3: Vertical Economy

To illustrate the role of input-output linkages, consider a vertical structure where Firm 1 produces final good using intermediate inputs from Firm 2 and labor: $Y_1 = A_1 Y_2^{\beta} L_1^{1-\beta}$, with $Y_2 = A_2 L_2$. The

[11] Interestingly, even in an efficient economy ($\mu_1 = \mu_2$), the contributions of each group through the income channel are non-zero: - $E_1 d\log Y_1 = -\lambda_1 d\log\mu_1 < 0$ - $E_2 d\log Y_2 = \lambda_1 d\log\mu_1 > 0$. This highlights the fact that the reallocation term captures not only allocative efficiency changes but reallocation of resources more broadly. These reallocation of resources in the aggregate do not matter in the case of an efficient economy.

two household groups have identical Cobb-Douglas preferences over both goods as in example 1 (α is the expenditure share of good 1). As in the previous examples, Household 1 owns labor, while Household 2 owns firms.

The factor share of labor in this economy is derived as follows:

$$\Lambda_L = \frac{\lambda_1}{\mu_1} + (1-\beta)\frac{\lambda_2}{\mu_2} \tag{8}$$

where λ_1 and λ_2 are the Domar weights of firms 1 and 2. Since Firm 1 uses Firm 2's output as an intermediate input, we have $\lambda_1 = \alpha + \beta(\lambda_2/\mu_2)$ and $\lambda_2 = 1-\alpha$. Substituting these relationships:

$$\Lambda_L = \frac{\alpha}{\mu_1} + \lambda_2\left[\frac{\beta}{\mu_1\mu_2} + \frac{(1-\beta)}{\mu_2}\right] = \frac{\alpha}{\mu_1} + \frac{(1-\alpha)}{\mathcal{M}_2} \tag{9}$$

where

$$\mathcal{M}_2 = \left[\frac{\beta}{\mu_1\mu_2} + \frac{(1-\beta)}{\mu_2}\right]^{-1} \tag{10}$$

The aggregate distortion is defined as:

$$\mathcal{M} = \frac{1}{\Lambda_L} = \left\{\frac{\alpha}{\mu_1} + \frac{(1-\alpha)}{\mathcal{M}_2}\right\}^{-1} \tag{11}$$

To interpret this structure, we need to consider the indirect use of labor in the vertical economy. The interpretation of $\mathcal{M}_2$ relates to the choice between using labor indirectly through Firm 1 or employing it directly when producing good 2.

When labor is used indirectly through Firm 1, the compound distortion is $\mu_1\mu_2$ because markup μ_2 is applied on top of the already marked-up price from Firm 1. In contrast, direct employment incurs only distortion μ_2. Regardless of the relative size of μ_1 and μ_2, the indirect use of labor in producing good 2 is always underutilized (since $\mu_1\mu_2 > \mu_2$). $\mathcal{M}_2$ can be interpreted as the harmonic mean of these distortions weighted by the intermediate input share parameter β.

At the level of $\mathcal{M}$, we have the harmonic mean of the distortion when Firm 1 sells to final consumers (μ_1) and the aggregate distortion of good 2 ($\mathcal{M}_2$), weighted by their respective shares in the economy. Depending on the value of μ_2, the distortion for Firm 1's sales to final consumers (μ_1) can be either smaller or larger than the aggregate distortion of good 2 ($\mathcal{M}_2$).

When Firm 1's output distortion increases ($d\log\mu_1 > 0$), the change in aggregate output is:

$$d\log Y = \tilde{\lambda}_1\left(\frac{\mathcal{M}}{\mu_1} - 1\right)d\log\mu_1 \tag{12}$$

where $d\log\Lambda_L = -(\mathcal{M}/\mu_1)\tilde{\lambda}_1 d\log\mu_1$.

When $\mathcal{M} > \mu_1$, an increase in μ_1 improves allocative efficiency ($d\log Y > 0$). While indirect employment is always underutilized in producing good 2, the distortion aggregated in $\mathcal{M}_2$ exceeds that of good 1, making good 1 overproduced. When Firm 1's output wedge increases, resources are reallocated to the underproduced good 2, improving allocative efficiency.

Following Proposition 1, the disaggregation into household contributions is:

$$
\begin{aligned}
d\log Y = \; & \underbrace{\left[\tilde{\lambda}_{11}\left(\frac{\mathcal{M}}{\mu_1}-1\right)d\log\mu_1\right]}_{-B_1 d\log P_1 \text{ (cost-of-living channel)}} + \underbrace{\left[-\frac{\mathcal{M}}{\mu_1}\lambda_1 d\log\mu_1\right]}_{dB_1 \text{ (income channel)}} \\
& + \underbrace{\left[\tilde{\lambda}_{21}\left(\frac{\mathcal{M}}{\mu_1}-1\right)d\log\mu_1\right]}_{-B_2 d\log P_2 \text{ (cost-of-living channel)}} + \underbrace{\left[\frac{\mathcal{M}}{\mu_1}\lambda_1 d\log\mu_1\right]}_{dB_2 \text{ (income channel)}}
\end{aligned}
\tag{13}
$$

With symmetric preferences, both households face the same price index change, so the cost-of-living channel is proportional to their expenditure shares. However, the magnitude of these effects, captured by $\mathcal{M}/\mu_1 - 1$, differs from the horizontal case due to the vertical structure.

The income channel works through changes in the labor share. The vertical linkages cause Λ_L to decrease more sharply than in a horizontal economy because the increased distortion at Firm 1 reduces labor demand both directly (through Firm 1's own production) and indirectly (through reduced demand for Firm 2's intermediate output). This results in a larger income transfer from labor-owning Household 1 to firm-owning Household 2.

The vertical economy thus demonstrates how production networks modify the distribution of reallocation across groups of households.

2.3 Ex-Ante Structural Analysis

We have focused so far on ex-post analysis that allows us to express outcomes as a function of sufficient statistics and minimal structural parameters. In particular, the results rely on changes in factor shares, $d\log\Lambda$, and final expenditure share changes, dB_g. These objects are endogenous and determined in equilibrium as shown in the examples of Section 2.2.

To express these factor share and final expenditure changes as a function of changes in primitives of the model, such as productivity and distortions, one needs to add more assumptions on preferences and technology and have additional information for measurement. The strategy we used in Section 2.1 to disaggregate the economy into the role played by different groups of households can be extended for ex-ante analysis. Doing this would allow not only to measure but also to predict changes and implement counterfactuals. One important challenge is the data requirement to implement ex-ante analysis in this context, which is typically more detailed than is standard, e.g., Atkin et al. (2025). For example, to fully pin down dB_g, one would need information about how firm profit changes affect household incomes through ownership shares. However, for ex-post analysis, it is sufficient to measure the observed dB_g. This captures the changes in consumption that are relevant for disaggregating the economy.

2.4 Application: Disaggregated Growth Accounting

In this section we explain how the results derived from Section 2.1 can be used for growth accounting. The appropriate measure of aggregate TFP growth, $\Delta \log Y_t - \sum_{f\in\mathcal{F}} \tilde{\Lambda}_{f,t-1}\Delta \log L_{f,t}$, is the part of growth in aggregate output that cannot be attributed to the pure technology effect of the growth of factors, and is weighted by the cost-based Domar weight of each factor, $\tilde{\Lambda}_f$. We now extend the result in Baqaee and Farhi (2020) to show how different groups of the economy account for pure changes in technology and reallocation.

Proposition 3. ***(Disaggregated Growth Accounting in Closed Economies)*** *Consider a partition of final demand into G groups indexed by g. The change in TFP in response to productivity shocks, factor supply shocks, and distortion shocks can be summarized, to a first-order, as:*

$$\underbrace{\Delta \log Y_t - \sum_{f\in\mathcal{F}} \tilde{\Lambda}_{f,t-1}\Delta \log L_{f,t}}_{\Delta\ Agreggate\ TFP} = \sum_{g\in\mathcal{G}} \underbrace{\left(\sum_{i\in\mathcal{N}} \tilde{\lambda}_{gi,t-1}\, d\log A_i\right)}_{\Delta\ Technology\ of\ Group\ g}$$
$$+ \sum_{g\in\mathcal{G}} \underbrace{\left(-\sum_{f\in\mathcal{F}} \tilde{\Lambda}_{gf,t-1} d\log \Lambda_f - \sum_{i\in\mathcal{N}} \tilde{\lambda}_{gi,t-1} \sum_{j\in\mathcal{N},\mathcal{F}} \tilde{\Omega}_{ij} d\log \tau_{ij}\mu_i + dB_g\right)}_{\Delta\ Reallocation\ of\ Group\ g}$$

The change in aggregate TFP can be decomposed into two terms. First, by technological changes driven by A. Second, by reallocation. Both of these terms are decomposed into different groups of final demand. The intuition of these terms follows the same logic as the ones described for Proposition 2.

2.5 Open Economy

In this section we extend the results from Section 2.1 to an open economy both in terms of imports and exports. We consider the case of a small and open economy.[12]

Similar to our approach in earlier sections, we partition final demand in the open economy into G disjoint groups of final demand, each indexed by g. Within this partition framework, export demand can be treated as another group of households.

Given the small open economy assumption, we treat imported inputs as primary production factors. Thus, the economy is populated by F^D domestic primary factors and F^M imported inputs

[12]It is straightforward to extend the theory to large economies. We do not consider this in our benchmark analysis as we are not aware of microdata to measure such a theory, which is the ultimate goal we are after. Also, a small and open economy is not a bad approximation since we apply the theory to the case of Chile.

such that $F = F^D + F^M$. Each group g faces the budget constraint:

$$\sum_{i\in\mathcal{N}} p_i y_{gi} = \sum_{f\in\mathcal{F}^D} w_f L_{gf} + \sum_{i\in\mathcal{N}} \pi_{gi}$$

where y_{gi} is group g's final demand of good i, L_{gf} is the quantity of factor f owned by group g, and π_{gi} is profits of firm i owned by group g.

Given that we treat imports as factors given by the rest of the world, the factor income associated with imports does not appear in the household budget constraints, as it is attributed to foreign households.[13] Note that this budget constraint implies that households cannot import final goods directly. All imports are channeled through intermediate inputs. This assumption comes from a feature of the data: final goods imports are also mostly channeled through intermediaries (either retail or wholesale). Thus, this assumption is without loss relative to the data because the model will feature a retail and a wholesale sector.

Using again the expenditure approach, GDP is the sum of final demand minus imports. Then GDP can be expressed as:

$$GDP = \sum_{g\in\mathcal{G}} E_g - \sum_{f\in\mathcal{F}^M} w_f L_f$$

GDP shares for each final output are described by the following vector:

$$b_i = \begin{cases} \frac{p_i y_i}{GDP} & \text{if } i \in \mathcal{N} \\ -\frac{w_i L_i}{GDP} & \text{if } i \in \mathcal{F}^M \\ 0 & \text{otherwise} \end{cases}$$

We treat export demand as a group in the partition of final demand in $\mathcal{G}$. Thus, we can maintain the same structure of disaggregated cost-based Domar weights as in the closed economy case.

Given this, we extend Proposition 3 to an open economy.[14]

Proposition 4. ***(Disaggregated Growth Accounting in Open Economies)*** *Consider a partition of final demand into G groups, each indexed by g. The change in TFP in response to productivity shocks, factor supply shocks, and distortion shocks can be summarized, to a first-order, as:*

$$\underbrace{\Delta \log Y_t - \sum_{f\in\mathcal{F}^D} \tilde{\Lambda}_{f,t-1} \Delta \log L_{f,t}}_{\Delta\ \textit{Agreggate TFP}} = \underbrace{\sum_{g\in\mathcal{G}} \left(\sum_{i\in\mathcal{N}} \tilde{\lambda}_{gi,t-1}\, d\log A_i + \sum_{f\in\mathcal{F}^M} \left(\tilde{\Lambda}_{gf,t-1} - \Lambda_{gf,t-1}\right) d\log L_f \right)}_{\Delta\ \textit{Technology of Group } g}$$

[13]We assume that profits are accrued to the supplier of the good, and therefore profits associated with imported goods are income to the foreign household. This affects how international trade-related distortions impact the domestic economy. This is a standard assumption in the literature (Atkin and Donaldson, 2022).

[14]It is direct to extend also Proposition 2 to an open economy. We focus on extending Proposition 3 because it is the main result we take to the data.

$$+\sum_{g\in\mathcal{G}}\underbrace{\left(-\sum_{f\in\mathcal{F}}\tilde{\Lambda}_{gf,t-1}d\log\Lambda_f-\sum_{i\in\mathcal{N}}\tilde{\lambda}_{gi,t-1}\sum_{j\in\mathcal{N},\mathcal{F}}\tilde{\Omega}_{ij}d\log\tau_{ij}\mu_i+dB_g\right)}_{\Delta\ \textit{Reallocation of Group g}}$$

Relative to the closed economy, there are two differences. First, technology has one additional component. This reflects the fact that imported intermediates are netted out of GDP using their cost rather than their marginal revenue product. In an economy with distortions, the gap between the cost and the marginal revenue product is captured by $\tilde{\Lambda}_{gf}-\Lambda_{gf}$. If $\tilde{\Lambda}_{gf}>\Lambda_{gf}$, distortions are positive. Thus, an increase in imported materials will increase domestic production (at constant prices) by more than imports (at constant prices), and hence an increase in L_f of imported factors f raises real GDP.

Second, the reallocation term is structurally similar to the closed-economy case, though with some additional components. First, the term $\sum_{f\in\mathcal{F}}\tilde{\Lambda}_{gf,t-1}d\log\Lambda_f$ now includes import shares. Since we treat imports as factors, when resources reallocate away from imports to underproducing parts of the economy just as with other factors, import shares decline and aggregate TFP improves. This captures reallocation effects similar to other factors. Changes in import shares also affect the income channel through their impact on $\sum_g B_g$.[15]

3 Data

We use data from five different administrative sources of the Chilean IRS (Servicio de Impuestos Internos, SII). One of the advantages of IRS data is that firms and workers have a unique tax identifier which allows us to merge individuals and firms across sources.

The first source used is the value-added tax form number 29. This form includes information about total sales, total materials expenditures, total imports, total exports, investment and main industry of the firm. This industry classification is at the 6-digit ISIC (rev. 4) level, which represents 626 sectors. This form covers all formal firms in the economy.

Second, we use the tax form number 1887, which has information about employer-employee relationships. Specifically, firms report all their payments to individual workers: the sum of taxable wages, overtime, bonuses, and any other labor earnings for each fiscal year. Since all legal firms must report to the SII, the data covers the total labor force with a formal labor contract, representing roughly 75% of employment in Chile. This form allows us to measure the total employment and total wage bill of firms.

Third, we use the income tax form number 22, which gathers yearly information on firms' balance sheets. This form covers all formal firms in the economy. This form provides data on

[15]Note that when we sum over all groups g, this result converges to Proposition 1 in Baqaee and Farhi (2019). By adding and subtracting import share changes, we can rewrite the reallocation term to obtain $\sum_{f\in\mathcal{F}^M}\left(\tilde{\Lambda}_{gf,t-1}-\Lambda_{gf,t-1}\right)d\log\Lambda_f$. Since $\sum_g B_g-\sum_{f\in\mathcal{F}^M}\Lambda_f=1$ in the open economy, aggregating our disaggregated decomposition recovers their aggregate formulation.

fixed assets to measure the capital stock of the firm using perpetual inventory methods. As initial condition, we use the first value of fixed assets reported by the firm in form number 22 and then investment from the tax form number 29 to update the capital stock each year. The real user cost of capital is built using publicly available data. We use the 10-year government bond interest rate minus expected inflation plus the external financing premium. Also, we use the capital depreciation rate from the LA-Klems database.[16]

Fourth, we use data from buying and selling books (forms number 3327-3328 and form 3323) for 2005-2014. This data provides information on firm-to-firm transactions.

Fifth, we use data from electronic tax documents that provide information on each product, including its price, traded domestically or internationally with at least one Chilean firm as a buyer or supplier from 2014 onwards. We use it to complement the buying and selling books to build the production network for the whole 2005-2023 period.

The data is anonymized to ensure confidentiality regarding firms' and workers' identities. A set of filters is applied over the raw data to obtain the final dataset for the quantitative analysis. First, for the complete data set, a firm is defined as active in a particular year if it has a tax ID, positive sales, materials, wage bill, and capital. We assume this given that all of these dimensions are necessary for estimating the production function of firms. Second, firms that have only one worker in a given year are dropped. This is to avoid tax structures that are not necessarily used for productive purposes.

To understand the representativeness of our sample, Table 1 displays the number of firms and value-added across sectors for the entire sample of the F29 form and our final sample. The firms in the final sample represent 85% of value added of the Chilean economy in 2022.

4 Measurement

To measure which parts of the economy drive aggregate productivity changes we focus on applying our results to growth accounting for an open economy as characterized by Proposition 4. As established in that proposition, to do disaggregated growth accounting in an open economy with distortions one needs to measure three sets of objects: (1) distortions $\tau_{ij}\mu_j$, (2) cost-based Domar weights $\tilde{\lambda}_{gi}$ and $\tilde{\Lambda}_{gi}$ and (3) aggregates such as value-added, factor share changes and expenditure share changes. We discuss each of them in turn.

4.1 Distortions

We measure distortions following the production approach presented by Hall (1988) and popularized by De Loecker and Warzynski (2012) which exploits the first-order condition of firms'

[16] For reference, the average government bond interest rate over the 2005-2022 period is 5.74%, the expected inflation is 4.6%, the external financing premium is 110 basis points and the average capital depreciation rate is 5%.

cost minimization problem. This approach implies that distortions can be expressed as the ratio between output elasticities and revenue input shares:

$$\tau_{ij}\mu_j = \frac{\sigma_{ij}}{\Omega_{ij}}, \tag{14}$$

where σ_{ij} is the elasticity of firm i's output with respect to input j, whether it is supplied by a factor or a firm. We use the first-order conditions of the cost minimization problem of all inputs, that is, labor, capital and materials. Each one of these will have potentially a different wedge for each firm. Thus, following Atkin et al. (2025), we do not rely on focusing only on variable inputs as the literature typically does (De Loecker and Warzynski, 2012), thereby allowing wedges to rise from other sources of distortions.[17]

As revealed by Equation (14), for measuring distortions we need two objects. Input shares of revenue and output elasticities. Input shares of revenue are measured directly from the data given by the definition in Section 2. To estimate output elasticities, we use standard estimation strategies from the industrial organization literature. Since we use a standard estimation strategy from the literature, we relegate the estimation details of distortions to the Appendix C. We only discuss here some implementation details regarding production function estimation.

To estimate output elasticities, we need to specify the technology and estimate production functions. We assume a Cobb-Douglas production function with time-invariant output elasticities. We estimate the production function separately for each 6-digit sector (which represents 626 sectors) that has at least 100 observations during our sample to recover output elasticities. Following Foster et al. (2022), we allow output elasticities to vary as much as possible by using the most disaggregated sector classification available in Chile. The firm-year observations belonging to sectors with more than 100 observations represent 97% of the sample. For the remaining sample for which we do not have enough observations, we estimate sectoral output elasticities at a higher level of sectoral aggregation: 160 sectors. We leverage the transaction-level firm-to-firm price data to estimate the production function using real, rather than nominal, inputs and outputs with price indices at the firm level.

4.2 Cost-Based Domar Weights and Final Demand Groups

The second object necessary to do disaggregated growth accounting in open economies with distortions, according to Proposition 4, is cost-based Domar weights,

$$\tilde{\lambda}_{gi} = \sum_{k \in \mathcal{N}} b_{gk} \tilde{\Psi}_{ki} = \sum_{k \in \mathcal{N}} b_{gk} (I + \tilde{\Omega}_{ki} + \tilde{\Omega}_{ki}^2 + \dots). \tag{15}$$

[17] Equation (14) identifies how to measure $\tau_{ij}\mu_j$ but not separately τ_{ij} from μ_j. For our application of disaggregated growth accounting, it is not necessary to separate the two. The literature typically identifies μ_j by assuming that one input is undistorted (De Loecker and Warzynski, 2012).

For this, we need to measure final expenditure shares b and the cost-based input-output matrix $\tilde{\Omega}$. We group households according to their final demand. We split their final demand into bins interacting the dimensions of sector, firm location, firm size and export status. That is, we implement the partition according to the products households purchase. This partition is easy to implement given the data that is typically used for doing growth accounting, which comes from firms' tax forms. It is an arbitrary partition, but it is a natural benchmark given that sectors, firm location, firm size, and export status are typical dimensions of analysis in macroeconomics and international trade.[18]

The dimensions of sectors and location do not vary significantly over time for each firm. Thus, firms belong only to one sector-location pair.[19] We consider 11 1-digit sectors, and split space into the 16 official regions of Chile. The dimension of export status allows for 2 states: selling domestically and exporting. For firms that perform both activities, we split them into separate units producing domestic and foreign sales. We consider 3 groups of firm size (small, medium, and large according to total sales) and we include a group to account for inactive firms.[20] Finally, we allow for the dimension of export status and firm size to change over time. We allow for this because transitions between firm sizes and export status are salient features of the data. We consider export status and these 4 firm size groups in 3 moments in time: at the beginning of the sample in 2005, at the peak of growth in the sample (2010), and at the end of the period of analysis (2022). We pick these moments to capture the expansion and stagnation of the Chilean economy between 2005 and 2022. Thus, we end up with, potentially, $11 \times 16 \times (4^3) \times (2^3) = 90,112$ groups.[21] Note that, by allowing for time-varying groups, we can incorporate the role of firm dynamics in decomposing aggregate productivity.

Final expenditure shares b is a vector of dimension $(N^D + N^X + F) \times 1$, where N^D is the number of domestic firms, N^X is the number of firms that export, F is the number of factors.[22] We measure three factors: labor, capital and imported intermediate inputs. Thus, $F = 3$ and $\mathcal{F} = \{L, K, M\}$.

The first N^D entries of b are measured by taking the residual between firms' total sales (excluding exports) and firms' intermediate sales to other firms (which we measure from the firm-to-firm data). This is the theory-consistent measure for final expenditures. For the next N^X entries, we measure them directly using firms exports given that all exports are considered final sales. The entries $N^D + N^X + 1$ and $N^D + N^X + 2$ correspond to labor and capital shares on final consumption,

[18]The decomposition presented in Proposition 4 could also be implemented by decomposing households according to their demographics. For implementing that, we would need to have data on changes in consumption for each demographic group, which is typically relatively less available.

[19]Although some firms might change sector and location over our sample period, we use the median sector and location of each firm over time given that these changes are rare.

[20]The sales cutoffs for these groups are the official ones used by the tax authority and they correspond approximately to the 64 and 94 percentile of the sales distribution of firms. Thus, for instance, 64 percent of firms are small.

[21]These are the potential number of groups because, in many cases, interactions between the dimensions that define groups are empty.

[22]We highlight explicitly the role of exporting firms because they are measured in a slightly different way than firms that sell domestically.

respectively. Since households do not buy domestic factors directly, they are zero. The last entry corresponds to imported materials expenditure, which enters with a negative sign since imports are discounted from GDP as shown in Section 2. All entries are then divided by GDP.

We measure the input-output matrix $\tilde{\Omega}$ at the firm-to-firm level using the firm-to-firm datasets and factor expenditures. The dimensions of $\tilde{\Omega}$ are $(N^D + N^X + F) \times (N^D + N^X + F)$ and is composed of different blocks:

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{DD} & \tilde{\Omega}_{DX} & \tilde{\Omega}_{DF} \\ \tilde{\Omega}_{XD} & \tilde{\Omega}_{XX} & \tilde{\Omega}_{XF} \\ \tilde{\Omega}_{FD} & \tilde{\Omega}_{FX} & \tilde{\Omega}_{FF} \end{bmatrix}$$

Since factors do not require inputs, the last row of matrices is zero, $\tilde{\Omega}_{Fj} = 0$ for all $j = \{D, X, F\}$. The fact that exports are sold only internationally implies that $\tilde{\Omega}_{DX} = 0$ and $\tilde{\Omega}_{XX} = 0$. For the remaining blocks, $\tilde{\Omega}_{DD}$, $\tilde{\Omega}_{XD}$, $\tilde{\Omega}_{DF}$, and $\tilde{\Omega}_{XF}$, we start with the corresponding revenue-based input-output matrix, Ω. The numerator of Ω_{DD} and Ω_{XD} is measured using the domestic firm-to-firm matrix and corresponds to the trade flow between firms. The numerators of Ω_{DF} and Ω_{XF} correspond to factor expenditures used in producing domestic goods and exports.[23] The denominators of Ω_{DD}, Ω_{XD}, Ω_{DF}, and Ω_{XF} correspond to buyers' total sales. To go from the revenue-based to the cost-based input-output matrix, we use $\tilde{\Omega}_{ij} = \tau_{ij}\mu_j\Omega_{ij}$. Specifically, for the matrices $\tilde{\Omega}_{DD}$ and $\tilde{\Omega}_{XD}$ we use the first-order condition of materials and for the matrices $\tilde{\Omega}_{DF}$ and $\tilde{\Omega}_{XF}$ we use the first-order conditions of labor, capital and imported inputs (which is the same one as materials given that domestic and foreign intermediate inputs enter as perfect substitutes within the materials bundle).

One important feature behind $\tilde{\lambda}_i$ is the role played by international trade. One can decompose $\tilde{\Psi}$ into block matrices similar to how we did with $\tilde{\Omega}$. Since imports are a factor in this economy, international trade plays a role on $\tilde{\Psi}_{DF}$ and $\tilde{\Psi}_{XF}$. Furthermore, these matrices can be decomposed into different matrix blocks that represent the role played by each factor. For instance, for the role that factors play in the production of goods sold domestically, we have,

$$\tilde{\Psi}_{DF} = \begin{bmatrix} \tilde{\Psi}_{DL} & \tilde{\Psi}_{DK} & \tilde{\Psi}_{DM} \end{bmatrix}.$$

The entries of $\tilde{\Psi}_{DM}$ measure the relevance that imports have in producing goods sold domestically, both directly and indirectly. A similar argument holds for exports through $\tilde{\Psi}_{XM}$. As a benchmark, we compare those matrices relative to the direct role that imports have in importers' costs, measured by $\tilde{\Omega}_{DM}$ and $\tilde{\Omega}_{XM}$, for both domestic producers and exporters, respectively.

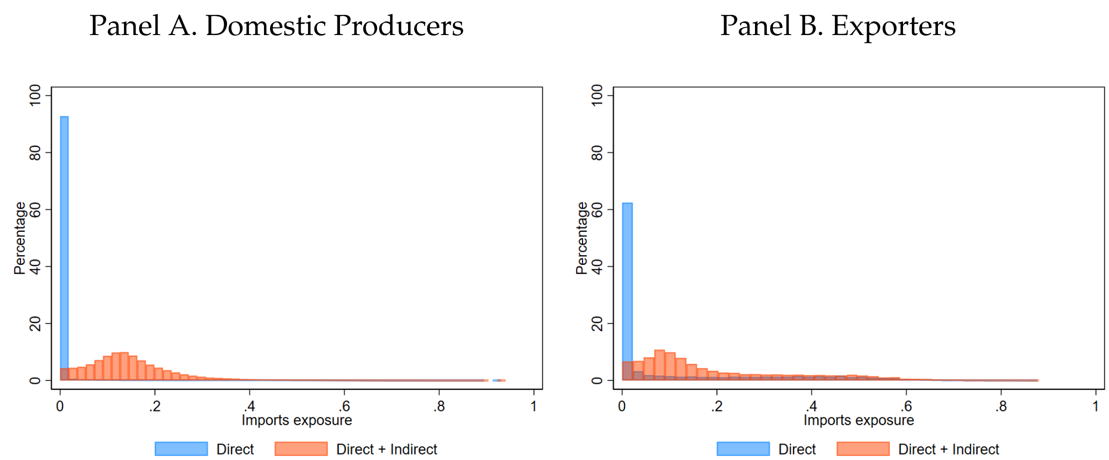
Figure 1 shows patterns behind $\tilde{\Psi}_{DM}$ and $\tilde{\Psi}_{XM}$, and $\tilde{\Omega}_{DM}$ and $\tilde{\Omega}_{XM}$. Panel A shows the distribution of the exposure of domestic producers to imports, both directly ($\tilde{\Omega}_{DM}$) and indirectly

[23] For firms that sell domestically and export, we allocate intermediate inputs and factors to exporting in proportion to export share in total sales of those firms. Among exporters, the majority of firms also sell domestically.

through the production network ($\tilde{\Psi}_{DM}$). Direct exposure is concentrated in a small share of firms (11%), but the indirect exposure is distributed across the economy, i.e., 1% of firms have zero indirect exposure to imports. This shows how important it is to consider indirect exposure when measuring the impact of imports on domestic producers.

Panel B of Figure 1 shows the distribution of the exposure to imports of exporters, both directly ($\tilde{\Omega}_{XM}$) and indirectly through the production network ($\tilde{\Psi}_{XM}$). Here, the share of firms directly exposed to imports is relatively larger (38% of exporters). This shows that exporters rely relatively more on imports as they engage more intensively with international trade than domestic producers. Although not all exporters are directly exposed to imports, all of them are exposed indirectly, highlighting that even for firms that engage in international trade, the indirect exposure to imports is relevant.

Figure 1: Direct and Indirect Exposure to Imported Intermediate Inputs



Note. This figure compares the distribution of firms' exposure to imported inputs, both directly ($\tilde{\Omega}_{DM}$ and $\tilde{\Omega}_{XM}$, in blue) and indirectly through input-output linkages ($\tilde{\Psi}_{DM}$ and $\tilde{\Psi}_{XM}$, in orange), for domestic producers (Panel A) and exporters (Panel B).

Another important feature we highlight from Proposition 4 is the role played by the disaggregated cost-based Domar weights in terms of how much it informs about the relevance of a cell gi of the economy relative to more standard measures of size. We compare the direct relevance of gi for the household as measured by b_{gi} to the indirect relevance through the entire supply chain, through $\tilde{\lambda}^I_{gi}$,[24] defined as

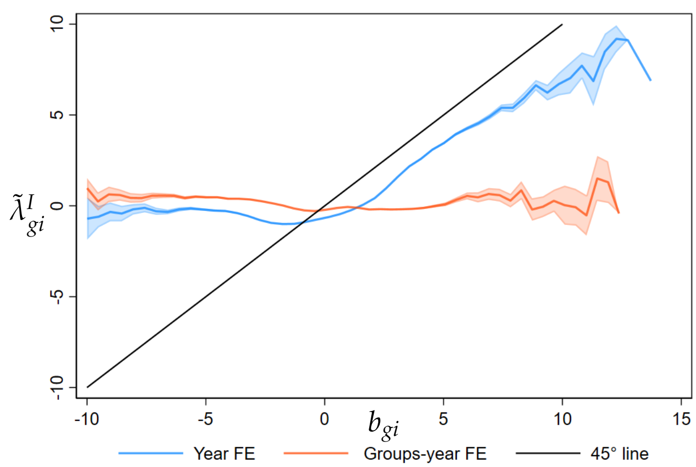
$$\tilde{\lambda}^I_{gi} = \sum_{k\in\mathcal{N}} b_{gk}(\tilde{\Omega}_{ki} + \tilde{\Omega}^2_{ki} + \ldots) \quad (16)$$

$$= \tilde{\lambda}_{gi} - b_{gi} \quad (17)$$

[24] Note that this measure excludes the direct relevance captured by b_{gi}.

Figure 2 correlates $b_{gi,t}$ with $\tilde{\lambda}^I_{gi,t}$ with non-parametric local linear regressions. Although there is a positive correlation, the relationship is non-linear. For cells gi below the median distribution of $b_{gi,t}$, there is no relationship between the two measures. For cells above the median, the correlation is positive and close to 1. This means that cells that are large in terms of direct exposure to households, are also large in terms of indirect exposure, whereas for cells that are small in terms of $b_{gi,t}$, the correlation is almost zero. This highlights how the direct exposure is a poor measure of relevance for aggregate output for some cells. To address how relevant the dimensions of sector, location, firm size and export status are for this relationship, we show the correlation after extracting fixed effects of interactions between all of those dimensions (interacted with year fixed). We find that the correlation between the two measures disappears. This implies that the direct exposure, $b_{gi,t}$ is relatively uninformative of the relevance of cells gi for aggregate output when we look at across firms within the dimensions we analyze.

Figure 2: Direct versus Indirect Cost-based Domar Weights: b_{gi} vs. $\tilde{\lambda}^I_{gi} = \tilde{\lambda}_{gi} - b_{gi}$



Note. This figure plots local linear regressions of the difference between cost-based Domar weights and direct exposure ($\tilde{\lambda}^I_{gi}$) against the direct exposure term b_{gi}. The blue is residualized from year fixed effects. The red curve is residualized from interacted fixed effects of sector, location, firm size, export status, and year.

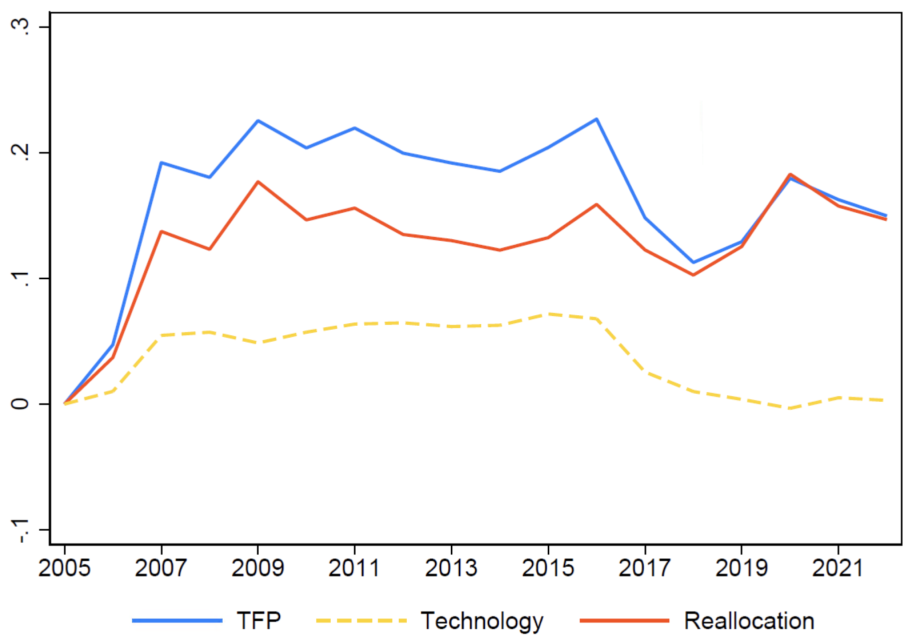
4.3 Aggregate Objects

Besides distortions and cost-based Domar weights, we need to measure aggregate objects to implement growth accounting according to Proposition 4. In particular, Y, L_L, L_K, L_M, Λ_L, Λ_K, Λ_M, B_g, which denote aggregate value added, employment, capital, imports and, labor, capital and import share, final expenditure shares, respectively. We measure Y, L_L, L_K and L_M as the sum of firms' value added, employment, capital and imports, respectively, across all firms in the economy. Factor shares of GDP, Λ_f, are measured as, total labor expenses, total capital times the user cost, and total import flows divided by GDP. Final expenditure shares, B_g, are measured as total sales minus intermediate sales from firm-to-firm transactions for each group g, divided by GDP.

5 Results

We start by showing the results of the main growth accounting exercise for open economies with distortions presented in Proposition 4. Figure 3 shows that aggregate TFP, measured by the distortion-adjusted Solow residual, exhibited strong growth between 2005 and 2009. After that, TFP has been stagnated, and even decreasing towards the end of the period. In fact, the TFP level in 2022 is lower than in 2009. The 2010s was a lost decade of aggregate productivity growth in Chile.[25]

Figure 3: Growth Accounting in Open Economies with Distortions
(cumulative percentage growth relative to 2005)



Note. Decomposition of aggregate TFP growth in Chile between 2005 and 2022 using the growth accounting framework for open economies with distortions presented in Proposition 4. The solid blue line shows aggregate TFP, measured as the distortion-adjusted Solow residual. The solid red line captures the reallocation term, defined as the contribution of firm-level reallocation of resources (inputs or sales) across heterogeneous producers. The dashed yellow line reflects the contribution of average firm-level productivity changes (technology).

Aggregate TFP growth between 2005 and 2022 is primarily driven by resource reallocation.

[25] These aggregate results are consistent with more standard measures of aggregate TFP growth that ignore both distortions and international trade (Central Bank of Chile, 2021).

The analysis so far could have been made using Baqaee and Farhi (2019). Nevertheless, under that analysis, reallocation is still a black box in terms of revealing which parts of the economy account for such a large share of the evolution of aggregate TFP growth.

To gain more insights into which segments of the economy drive the evolution of reallocation, we implement our disaggregation result using the partition of the economy described in Section 4. We focus on the disaggregation of reallocation since the evolution of technology matters quantitatively little.

Figure 4 shows the contribution of sectors and export status to the cumulative growth of reallocation for the period of TFP expansion (2005-2009) and stagnation (2010-2022).[26] TFP growth in the first period is driven by improvements in reallocation in mining, manufacturing, utilities and retail/wholesale, with retail/wholesale being the most relevant sector. Exports dominate reallocation improvements in mining. The majority of improvements in manufacturing, utilities and retail/wholesale are due to domestic activity. We further find that the cost-of-living channel dominates the expansion period since final expenditure shares changed little over this period. Most of the improvements in reallocation are driven by efficiency improvements that propagate through the supply chain thereby benefiting households through their exposure in consumption.

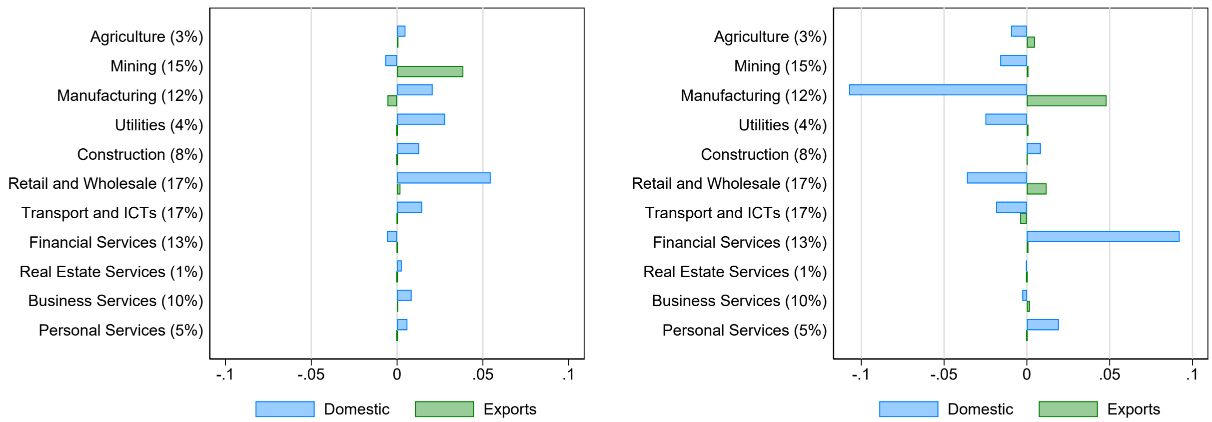
Note that Figure 4 also shows that sectoral value-added shares, a common statistic used to measure how important a sector is, is a poor measure of a sector's relevance for reallocation. For instance, although retail/wholesale and transport/ICT account for the same value-added shares, the former represents more than 3 times the relevance for reallocation relative to the latter.

The stagnation period is driven by reductions in reallocation of these same sectors, plus transport and ICTs. In particular, the reduction in reallocation of domestic output of manufacturing dominates. This reduction is compensated by the increase in reallocation of the export output of manufacturing and of the financial sector. Opposite to the expansion period, the stagnation period is driven not by the cost-of-living but the income channel. Final expenditure shares fell significantly for manufacturing and retail/wholesale as can be seen in Figure 5.

[26]By construction, summing across all these terms is equal to the total cumulative change of reallocation of each period reported in Figure 3.

Figure 4: Disaggregating Reallocation: Sectors and Export Status

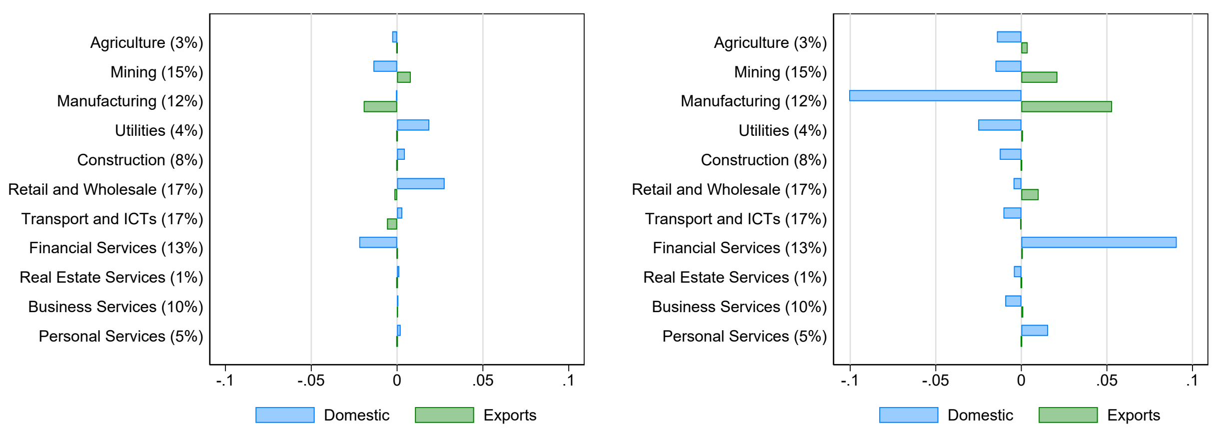
Panel A: Expansion Period (2005-2009) Panel B: Stagnation Period (2010-2022)



Note. Each bar shows the contribution of a sector's domestic and export activity to cumulative growth in reallocation, with average sector GDP shares in parentheses.

Figure 5: Changes in Final Expenditure Shares (dB_g): Sectors and Export Status

Panel A: Expansion Period (2005-2009) Panel B: Stagnation Period (2010-2022)



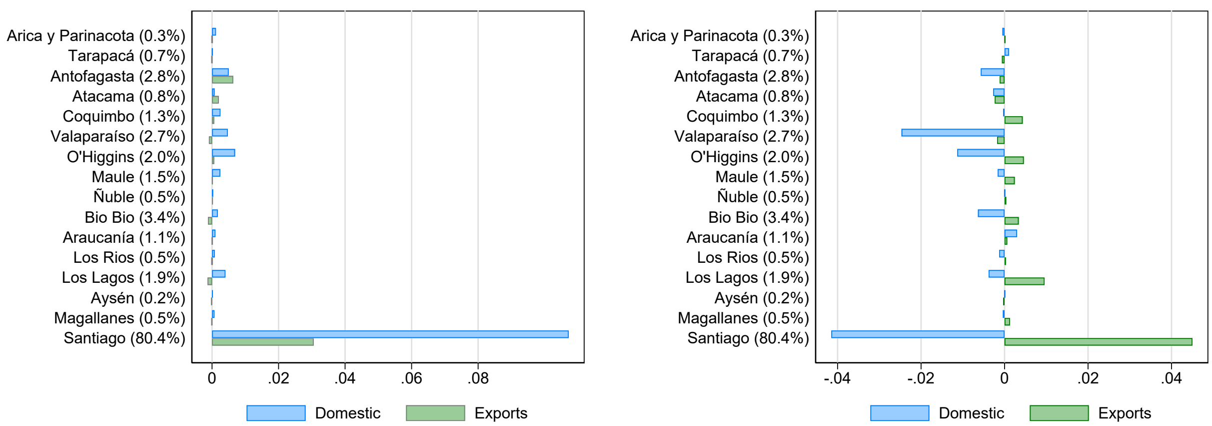
Note. Each bar shows changes of final expenditure shares (dB_g) on a sector's domestic and export activity, with average sector GDP shares in parentheses.

Figure 6 presents how regions and export status account for reallocation. The region of Santiago matters the most in the expansion period, in particular in the domestic side. In the stagnation period, Santiago matters less since the domestic and export side cancel each other out. Instead, the region of Valparaiso, O'Higgins and Bio Bio matter relatively more in the stagnation period, through the domestic side. This coincides with the reduction in manufacturing reallocation reported in Figure 4, since those regions rely significantly on manufacturing activity.

Figure 6: Disaggregating Reallocation: Geography and Export Status

Panel A: Expansion Period (2005-2009)

Panel B: Stagnation Period (2010-2022)

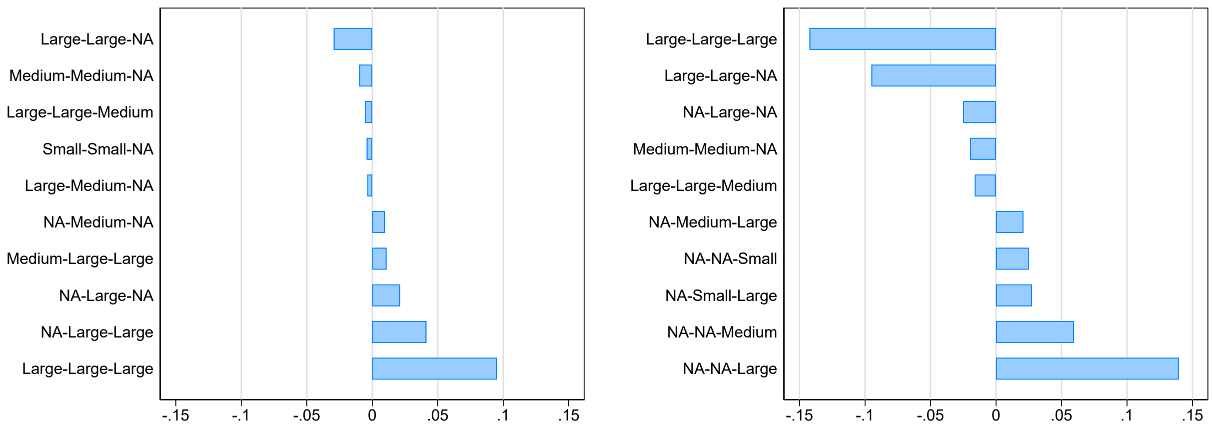


Note. Each bar shows a region's contribution to reallocation by domestic and export activity, with average sector GDP shares in parentheses.

Figure 7 disaggregates the evolution of reallocation by firm-size groups, where firm size is fixed based on their status in three distinct periods: 2005–2009 (expansion), 2010, and 2011–2022 (stagnation). During the expansion period, the largest reallocation gains come from groups dominated by firms that were already large in the first period (e.g., Large–Large–Large), underscoring the central role of established large firms in driving reallocation gains. By contrast, the stagnation period is marked by steep losses in reallocation among those same groups, particularly groups that remained large or became large by 2010, suggesting that reallocation stagnation is closely tied to declining performance among formerly dominant firms. Thus, firm entry and exit appears to explain little in the evolution of aggregate productivity.

Figure 7: Disaggregating Reallocation: Evolution of Firm Size

Panel A: Expansion Period (2005-2009) Panel B: Stagnation Period (2010-2022)

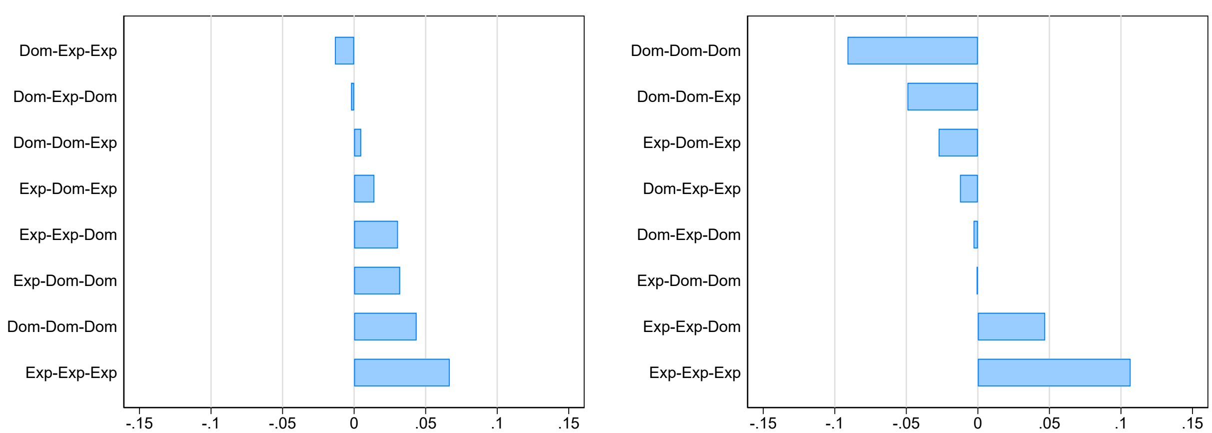


Note. Bars show the top five and bottom five contributions to reallocation by firm-size group, across three periods: 2005–2009 (first), 2010 (middle), and 2011–2022 (last).

Figure 8 presents how the evolution of export status matters for reallocation. Firms that stayed exporters throughout the period are the ones that contribute the most to reallocation improvements whereas the ones that stayed selling domestically contribute the most to the stagnation of reallocation.

Figure 8: Disaggregating Reallocation: Entry and Exit into Export Status

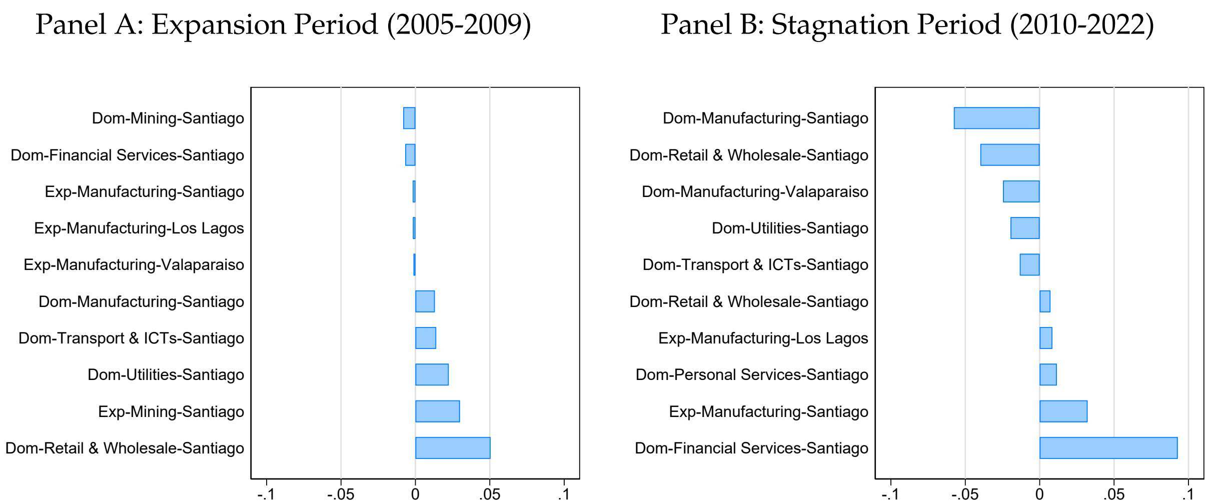
Panel A: Expansion Period (2005-2009) Panel B: Stagnation Period (2010-2022)



Note. Each bar shows the top five and bottom five contributions to reallocation by the evolution of export status.

Figure 9 presents how the interaction of sectors, regions and export status matter for reallocation. Since the combination of these groups are too many, this figure presents the top and bottom 5 biggest groups in terms of their contribution to reallocation each period. Among these, the group that accounts for the largest increase in reallocation during the expansion period is domestic side of retail/wholesale of Santiago, whereas the group that accounts the most in the stagnation period is domestic side of manufacturing from Santiago.

Figure 9: Disaggregating Reallocation: Sectors, Geography and Export Status



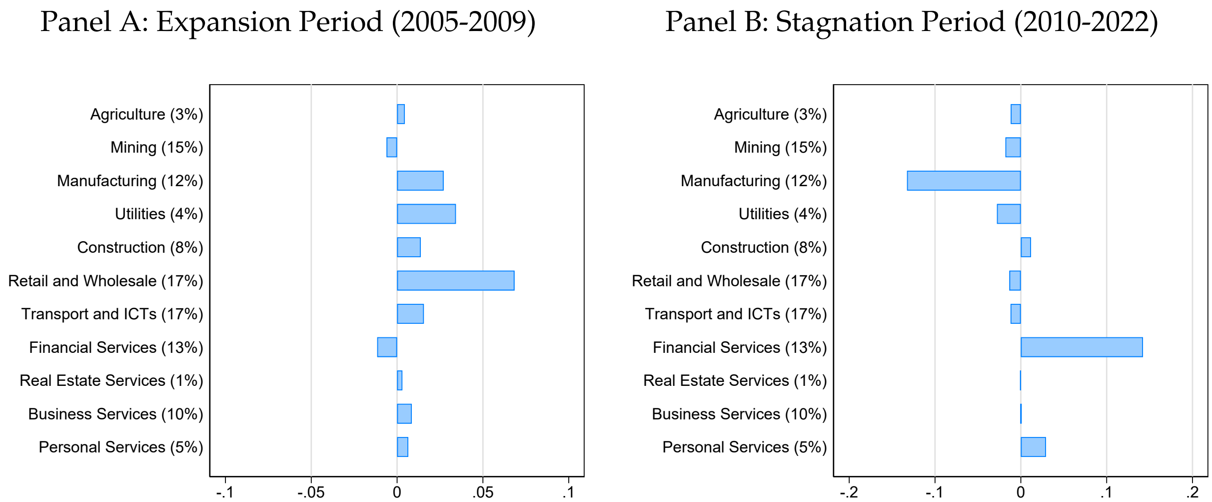
Note. Each bar shows the top five and bottom five contributions to reallocation by sector, region, and export status group.

5.1 Robustness: Closed Economy, Material Wedges, and Industry Aggregation

We present three robustness of counterfactual models to understand the drivers of the results presented in the previous section. First, we shut down international trade and implement the growth accounting decomposition from Proposition 3.[27] Figure 10 replicates Figure 4 for a closed economy. Mining matters now little for both the expansion and stagnation period and retail now does not reduce reallocation during the stagnation. This highlights how important is international trade in accounting for the reallocation of mining, whereas it matters little for the role of manufacturing.

[27]To implement this robustness while keeping aggregates fixed, we distribute import expenses among domestic suppliers of importing firms and distribute exports among domestic buyers of exporting firms.

Figure 10: Disaggregating Reallocation in a Closed Economy: Sectors and Exports



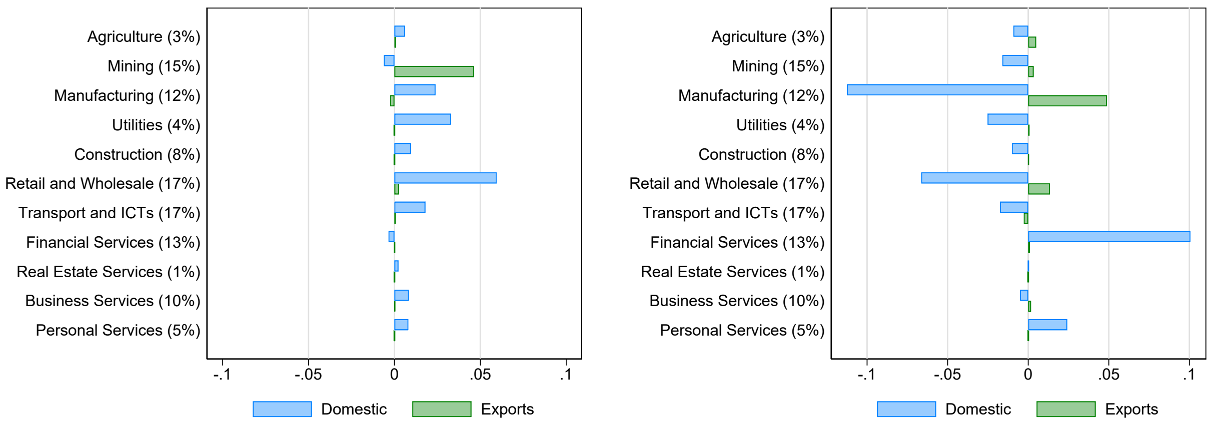
Note. This figure replicates the sectoral decomposition of reallocation under a closed economy counterfactual that eliminates exporting goods and imported materials. Each bar shows a sector's contribution to reallocation.

As a second counterfactual robustness, we implement an economy with only material wedge and thus $\tau_{ij}\mu_j = 1$ for transactions involving labor and capital factors.[28] Figure 11 shows that, relative to the baseline results, retail/wholesale contributes much more to the stagnation of reallocation. This implies that changes in labor and capital wedges contributed positively to the reallocation of retail/wholesale.

[28]For this counterfactual we recalibrate $\tilde{\Omega}$ so that identities of the theory still hold.

Figure 11: Disaggregating Reallocation Only Material Wedge: Sectors and Exports

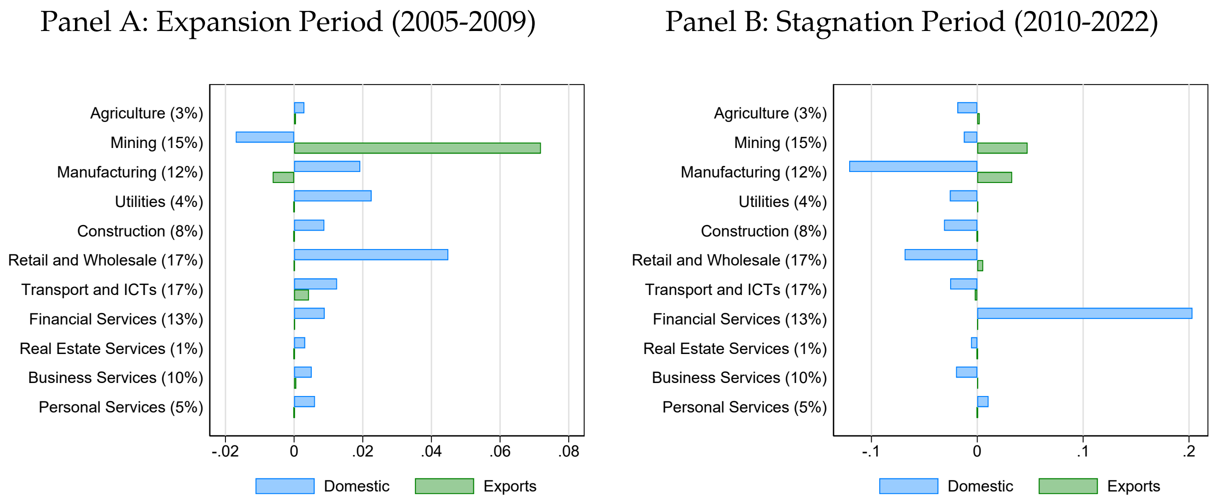
Panel A: Expansion Period (2005-2009) Panel B: Stagnation Period (2010-2022)



Note. This figure shows sectoral contributions to reallocation under a counterfactual where only material input distortions are active; labor and capital wedges are muted. Sector GDP shares (2005–2022 averages) are shown in parentheses.

Finally, we implement an aggregated version of the analysis. Instead of using firm-level analysis, we use sectoral-level analysis. That is, we aggregate input-output matrices and all other firm variables at the sectoral level and recompute the entire analysis. Figure 12 shows that, relative to the baseline results, exporting activity of the mining sector matters significantly more for the expansion period whereas the domestic side of the financial sector contributes almost twice to reallocation in the expansion period. The overall magnitudes and the ranking of which groups in the economy matter for reallocation are substantially different when aggregating the economy at the sectoral level.

Figure 12: Disaggregating Reallocation at Sector-Level: Sectors and Exports



Note. This figure shows sector-level contributions to reallocation, using a version of the model aggregated at the sector level rather than at the firm level. Sector GDP shares (2005–2022 averages) are shown in parentheses.

6 Conclusion

Although understanding aggregate productivity has been shown to be fundamental for growth and development, it continues to be an elusive black box. Which disaggregated parts of the economy account for its evolution? This is a fundamental aspect that policymakers constantly debate and try to figure out, especially in countries swamped in secular stagnation. This debate occurs in the context of implementing growth accounting decompositions and also when discussing policies to overcome productivity stagnation, such as industrial policies.

It is understood that aggregate productivity evolves according to technology and changes in the allocation of resources. But which parts of the economy drive technology? Which parts drive reallocation? Under what conditions does a part of the economy contribute positively or negatively to aggregate productivity through the reallocation of resources? We present an aggregation result to answer these questions that structurally dissects technology and reallocation. Our main theoretical result presents an interpretable and structural decomposition of aggregate productivity, of how technology of different groups evolve and how resources are reallocated as a response to a shock. Changes in aggregate technology and reallocation can be decomposed into arbitrary groups of the economy, which in turn are functions of sufficient statistics of how factor shares, consumption shares and distortions change. This disaggregation result can be implemented with increasingly standard administrative datasets.

We apply our main theoretical result to revisit growth accounting. We use a comprehensive administrative firm-to-firm tax data for the universe of formal firms from Chile to measure the microeconomic drivers of aggregate productivity stagnation since the early 2010s. Aggregate productivity stagnation is almost entirely explained by reallocation. Export activity of mining, domestic activity of manufacturing and retail, and large firms shape the bulk of this stagnation. The relevance of different groups in driving this stagnation is explained by the income channel of how expenditures across households changed over this period. We show that ignoring the microeconomic details of our framework such as which distortions are included, the level of aggregation and whether international trade is taken into account are consequential for identifying the drivers behind aggregate productivity growth.

We identify two avenues of follow-up work. First, we focus on ex-post analysis, which takes changes in endogenous objects, such as factor and consumption shares, as given. Implementing ex-ante analysis is challenging when disaggregating reallocation because it is necessary to trace which parts of the economy ultimately benefit from the revenue that distortions generate. Pushing on this direction is important if one is to fully understand what drives aggregate reallocation and growth. Second, we limit our analysis to constant-returns-to-scale technologies and preferences. Extending our results to allow for flexible returns to scale would be useful given how scale economies can affect different households and firms in the economy, especially when thinking about long-run development questions.

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Appendix

A Proofs

The proof of Proposition 1 is in the main text. Since Propositions 2 and 3 are special cases of Proposition 4, we focus on the proof of Proposition 4.

Proof of Proposition 4

Proof. We start from the price equation. For all $i \in \mathcal{N}$:

$$d\log p_i = -d\log A_i + d\log\mu_i + \sum_{j\in\mathcal{N}} \tilde{\Omega}_{ij} d\log p_j + \sum_{j\in\mathcal{N},\mathcal{F}} \tilde{\Omega}_{ij} d\log\tau_{ij} + \sum_{f\in\mathcal{F}} \tilde{\Omega}_{if} d\log w_f,$$

In matrix notation, we have:

$$\begin{aligned} d\log p &= (I-\tilde{\Omega}^{N\times N})^{-1}\left[-d\log A + d\log\mu + \tilde{\Omega}^{N\times(N+F)} d\log\tau + \tilde{\Omega}^{N\times F} d\log w\right] \\ &= -(I-\tilde{\Omega}^{N\times N})^{-1}[d\log A - d\log\mu] + (I-\tilde{\Omega}^{N\times N})^{-1}\tilde{\Omega}^{N\times(N+F)} d\log\tau + (I-\tilde{\Omega}^{N\times N})^{-1}\tilde{\Omega}^{N\times F} d\log w_f \end{aligned}$$

where $\tilde{\Omega}^{N\times N}$ is the square matrix extracted for the first $N\times N$ of the cost-based IO matrix, $\tilde{\Omega}$. From the property of inverse matrix, $(I-\tilde{\Omega}^{N\times N})^{-1}$ is equal to the first $N\times N$ matrix extracted from cost-based Leontief inverse matrix, $\tilde{\Psi}$.

Therefore, we can express:

$$\begin{aligned} d\log p_i &= -\sum_{j\in\mathcal{N}} \tilde{\Psi}_{ij}[d\log A_j - d\log\mu_j] + \sum_{j\in\mathcal{N}}\sum_{k\in\mathcal{N},\mathcal{F}} \tilde{\Psi}_{ij}\tilde{\Omega}_{jk} d\log\tau_{jk} + \sum_{f\in\mathcal{F}} \tilde{\Psi}_{if} d\log w_f, \\ &= -\sum_{j\in\mathcal{N}} \tilde{\Psi}_{ij}[d\log A_j - \sum_{k\in\mathcal{N},\mathcal{F}} \tilde{\Omega}_{jk} d\log\left(\tau_{jk}\mu_j\right)] + \sum_{f\in\mathcal{F}} \tilde{\Psi}_{if}\left(d\log w_f L_f - d\log L_f\right) \end{aligned}$$

By definition of the GDP deflator, we know:

$$d\log P = \sum_{i\in\mathcal{N}} \frac{p_i y_i}{GDP} d\log p_i - \sum_{f\in\mathcal{F}^M} \frac{w_f L_f}{GDP} d\log w_f$$

So,

$$\begin{aligned} d\log P &= b' d\log p - \sum_{f\in\mathcal{F}^M} \Lambda_f\left(d\log w_f L_f - d\log L_f\right) \\ &= -\sum_{g\in\mathcal{G}}\sum_{i\in\mathcal{N}} \tilde{\lambda}_{gi}\left(d\log A_i - \sum_{j\in\mathcal{N},\mathcal{F}} \tilde{\Omega}_{ij} d\log\left(\tau_{ij}\mu_i\right)\right) \end{aligned}$$

$$+\sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}}\tilde{\Lambda}_{gf}\left(d\log\Lambda_f - d\log L_f\right) - \sum_{f\in\mathcal{F}}\Lambda_f\left(d\log\Lambda_f - d\log L_f\right)$$

Since we know $d\log Y = d\log GDP - d\log P$ and $d\log GDP = \sum_{g\in\mathcal{G}}\frac{E_g}{GDP}d\log E_g - \sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}^M}\Lambda_{gf}d\log\Lambda_f$:

$$\begin{aligned}
d\log Y &= d\log GDP - d\log P \\
&= \sum_{g\in\mathcal{G}}\frac{E_g}{GDP}d\log E_g - \sum_{f\in\mathcal{F}^M}\Lambda_f d\log\Lambda_f - \sum_{g\in\mathcal{G}}\sum_{j\in\mathcal{N}}\frac{p_i y_{gi}}{GDP}d\log p_i + \sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}^M}\Lambda_{gf}\left(d\log w_f L_f - d\log L_f\right) \\
&= \sum_{g\in\mathcal{G}}B_g\left(d\log E_g - \sum_{i\in\mathcal{N}}\frac{p_i y_{gi}}{E_g}d\log p_i\right) - \sum_{f\in\mathcal{F}^M}\Lambda_f d\log\Lambda_f + \sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}^M}\Lambda_{gf}\left(d\log w_f L_f - d\log L_f\right) \\
&= \sum_{g\in\mathcal{G}}B_g\left(d\log B_g\right) - \sum_{g\in\mathcal{G}}B_g\left\{\sum_{i\in\mathcal{N}}\frac{p_i y_{gi}}{E_g}d\log p_i/GDP\right\} - \sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}^M}\Lambda_{gf}d\log L_f \\
&= \sum_{g\in\mathcal{G}}dB_g - \sum_{g\in\mathcal{G}}\sum_{i\in\mathcal{N}}b_{gi}\left\{-\sum_{j\in\mathcal{N}}\tilde{\Psi}_{ij}[d\log A_j - \sum_{k\in\mathcal{N},\mathcal{F}}\tilde{\Omega}_{jk}d\log\left(\tau_{jk}\mu_j\right)] + \sum_{f\in\mathcal{F}}\tilde{\Psi}_{if}\left(d\log\Lambda_f - d\log L_f\right)\right\} - \sum_{f\in\mathcal{F}^M}\Lambda_f d\log L_f \\
&= \sum_{g\in\mathcal{G}}dB_g - \sum_{g\in\mathcal{G}}\left\{-\sum_{j\in\mathcal{N}}\tilde{\lambda}_{gj}\left[d\log A_j - \sum_{k\in\mathcal{N},\mathcal{F}}\tilde{\Omega}_{jk}d\log\left(\tau_{jk}\mu_j\right)\right] + \sum_{f\in\mathcal{F}}\tilde{\Lambda}_{jf}\left(d\log\Lambda_f - d\log L_f\right)\right\} - \sum_{f\in\mathcal{F}^M}\Lambda_f d\log L_f
\end{aligned}$$

Following Baqaee and Farhi (2020), we define distortion-adjusted TFP as:

$$\begin{aligned}
d\log TFP &\equiv d\log Y - \sum_{f\in\mathcal{F}^D}\tilde{\Lambda}_f d\log L_f, \\
&= d\log Y - \sum_{g\in\mathcal{G}}\sum_{f\in\mathcal{F}^D}\tilde{\Lambda}_{gf}d\log L_f
\end{aligned}$$

Therefore, rearranging the equation, we obtain:

$$\begin{aligned}
\underbrace{\Delta\log Y_t - \sum_{f\in\mathcal{F}^D}\tilde{\Lambda}_{f,t-1}\Delta\log L_{f,t}}_{\Delta\text{ Agreggate TFP}} &= \sum_{g\in\mathcal{G}}\underbrace{\left(\sum_{i\in\mathcal{N}}\tilde{\lambda}_{gi,t-1}\,d\log A_i + \sum_{f\in\mathcal{F}^M}\left(\tilde{\Lambda}_{gf,t-1} - \Lambda_{gf,t-1}\right)d\log L_f\right)}_{\Delta\text{ Technology of Group g}} \\
&+ \sum_{g\in\mathcal{G}}\underbrace{\left(-\sum_{f\in\mathcal{F}}\tilde{\Lambda}_{gf,t-1}d\log\Lambda_f - \sum_{i\in\mathcal{N}}\tilde{\lambda}_{gi,t-1}\sum_{j\in\mathcal{N},\mathcal{F}}\tilde{\Omega}_{ij}d\log\tau_{ij}\mu_i + dB_g\right)}_{\Delta\text{ Reallocation of Group g}}
\end{aligned}$$

□

B Data Cleaning and Representativeness

Table 1: Sample Statistics by Sector

	# of Tax IDs			Value Added (2022 million pesos)		
	Raw	Sample	Share sample	Raw	Sample	Share sample
Agriculture	105,010	27,821	0.26	5,884,980	4,394,102	0.75
Mining	5,042	1,570	0.31	12,603,017	12,439,233	0.99
Manufacturing	120,655	36,545	0.30	27,529,125	26,283,199	0.95
Utilities	7,085	2,047	0.29	5,489,654	4,125,247	0.75
Construction	120,981	30,589	0.25	16,053,036	10,985,617	0.68
Retail and Wholesale	487,486	124,238	0.25	28,774,760	23,163,400	0.80
Transport	148,385	33,428	0.23	29,541,936	24,449,458	0.83
Financial Services	42,222	8,513	0.20	47,436,467	45,477,105	0.96
Real Estate Services	23,034	7,041	0.31	3,732,899	1,986,035	0.53
Business Services	147,152	33,950	0.23	18,616,468	15,031,352	0.81
Personal Services	203,389	21,682	0.11	11,961,647	8,795,058	0.74
Total	1,410,441	327,424	0.23	207,623,989	177,129,806	0.85

Note. This table reports the number of firms (measured as unique tax IDs) and aggregate value added across 1-digit sectors in the economy. "Raw" refers to the full population of tax IDs filing in 2022, while "Sample" corresponds to the subset retained for empirical analysis after applying data quality filters and merging with transaction-level data. Value added is defined as firm revenue minus intermediate input expenditures. Overall, the sample retains 23% of firms and 85% of aggregate value added. Sectoral coverage varies: the six largest sectors in terms of value added all retain at least 80% of their original value added.

We document the coverage of our analytical sample relative to the full universe of firms. Table 1 shows that, while we retain only 23% of firms (measured as unique tax IDs), these firms account for 85% of aggregate value added in 2022. This reflects our focus on firms with complete and consistent information. Sectoral coverage is uneven in terms of firm counts but remains high in terms of economic weight: we retain at least 80% of value added in the six largest sectors. This indicates that, despite keeping less than one-quarter of firms, the sample remains representative of aggregate economic activity.

C Wedges Estimation Strategy and Results

We use an output-based Cobb-Douglas production function with three factors: capital (K), Labor (L), and Materials (M) to recover material-output elasticities where lowercase variables denote natural logarithms:

$$q_{it} = A_{it} + \beta_l \, l_{it} + \beta_k \, k_{it} + \beta_m + \epsilon_{it}$$

To ensure parameter identification, we draw upon Ackerberg et al. (2015). The sequence of decisions required for identification proceeds as follows: Capital is a state variable determined at period $t-1$. Labor can be selected between $t-1$ and t, but always after the capital decision and before the materials decision. While it is acknowledged that demand-side shocks can potentially impact markup measures (Doraszelski and Jaumandreu (2021)), addressing these concerns goes beyond the scope of this work.

To recover price variation-free variables for output and materials, we construct firm-level price indexes using standard Tornqvist indices. This method to build price indices is widely recognized for estimating aggregate production functions at the firm or plant level when price data is accessible (Dhyne et al. (2022) and De Roux et al. (2021)). This allows us to infer quantity-based instead of revenue-based output elasticities and avoid a common critique from the literature ((Bond et al., 2021)).

We compute firm-specific annual weighted average prices (P_{igt}) for each product (g) sold by firm i during year t. Subsequently, we construct firm-specific price indices (ΔP_{it}) for products observed in consecutive years using the product-level weighted average price and the share of the product present in both year $t-1$ and year t:

$$\Delta \log P_{it} = \sum_{g} \frac{s_{igt} + s_{igt-1}}{2} \, \Delta \log(P_{igt})$$

Where s_{igt} represents the revenue share of product g for firm i at time t. We perform an analogous procedure for materials used in production; consequently,

$$q_{it} \approx \frac{\text{Revenue}_{it}}{P_{it}} \quad ; m_{it} \approx \frac{\text{Material expenditure}_{it}}{P^M_{it}}$$

We conduct separate production function estimations for every 626 industries at the 6-digit level present on the Chilean IRS records. Our sample selection is contingent upon having a minimum of 100 observations in each sector. Building on the approach outlined in Foster et al. (2022), our objective is to allow output elasticities to exhibit as much variation as possible within the same aggregate industry.

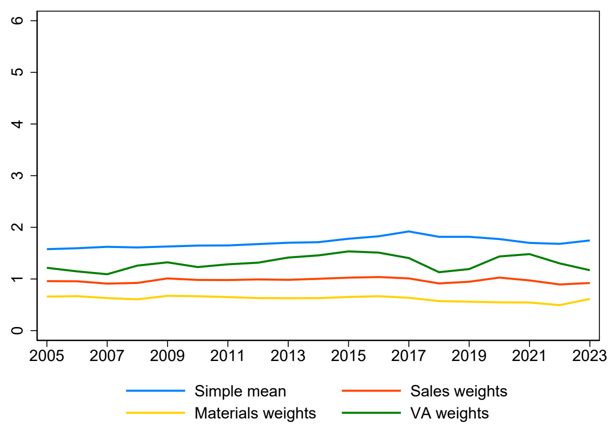
We successfully estimate production functions for 97% of firm-year observations within the 6-digit industries that meet the minimum data requirement. However, for the remaining 3% of firm-year observations, where data is insufficient, we extend our production function estimation to 160 sectors and 11 sectors.

As firm-level price data is only available for 2014-2022, we estimate the production function and recover input-output elasticities for this period. We assume time-invariant elasticities, which allows us to estimate wedges for the full 2005-2022 sample period. For years prior to 2014, we utilize available data on firm sales and input expenditures to construct input expenditure shares.

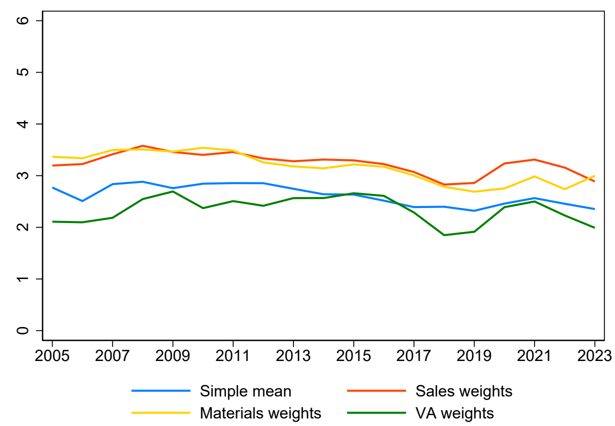
We document the evolution of wedges moments by a factor over time in Figure 13. Additionally, we illustrate sector heterogeneity in median wedges in Figure 14.

Figure 13: Wedges evolution in time

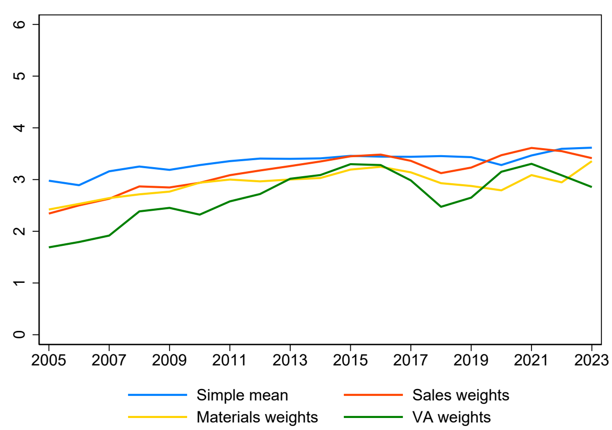
Panel A: Materials wedge



Panel B: Labor wedge

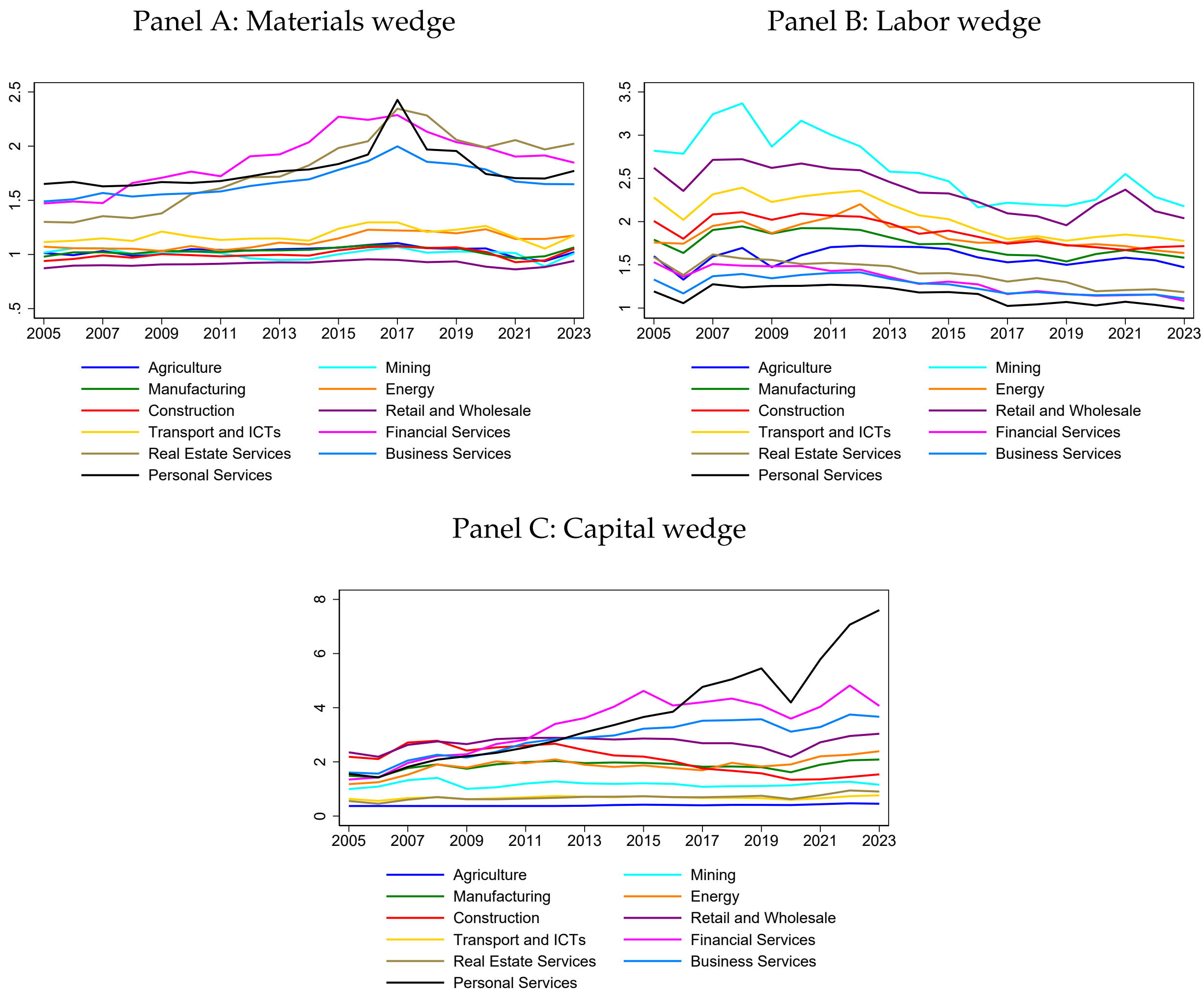


Panel C: Capital wedge



Note. This figure shows the time evolution of wedges in materials (Panel A), labor (Panel B), and capital (Panel C) inputs over the 2005–2022 period. Wedges are estimated using sector-specific production function elasticities assuming constant elasticities over time. Each wedge reflects the log deviation between actual and efficient input use.

Figure 14: Median Wedges evolution by sector



Note. This figure shows the sectoral heterogeneity in median wedges for materials (Panel A), labor (Panel B), and capital (Panel C) across 6-digit industries. Wedges are computed at the firm level using the production function approach and then aggregated by sectoral medians. Only industries with at least 100 observations are included. The variation across sectors highlights the uneven incidence of input distortions within the Chilean economy.

To unpack both components of distortions, in Table 2, we present the median material-output elasticities by 11 sectoral aggregations, while in Figure 15 we show the median factor revenue shares evolution by sector.

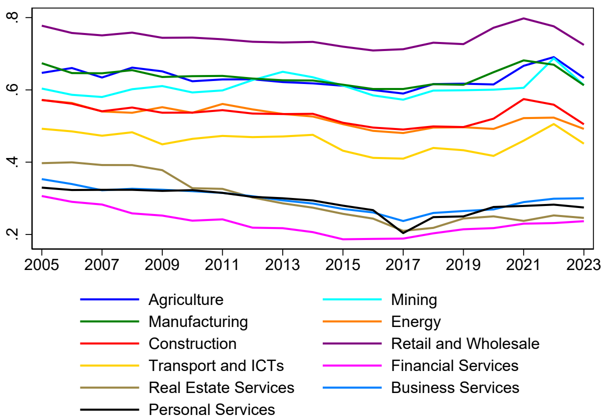
Table 2: Elasticities and Returns to Scale by Sector

Sector	Materials	Labor	Capital	RTS
Agriculture	0.673	0.304	0.031	1.008
Mining	0.634	0.373	0.048	1.055
Manufacturing	0.648	0.393	0.030	1.071
Utilities	0.544	0.448	0.067	1.059
Construction	0.544	0.471	0.032	1.047
Retail and Wholesale	0.572	0.565	0.030	1.167
Transport and ICTs	0.530	0.432	0.030	0.992
Financial Services	0.425	0.571	0.031	1.027
Real Estate Services	0.523	0.469	0.024	1.016
Business Services	0.485	0.482	0.026	0.993
Personal Services	0.515	0.469	0.026	1.010

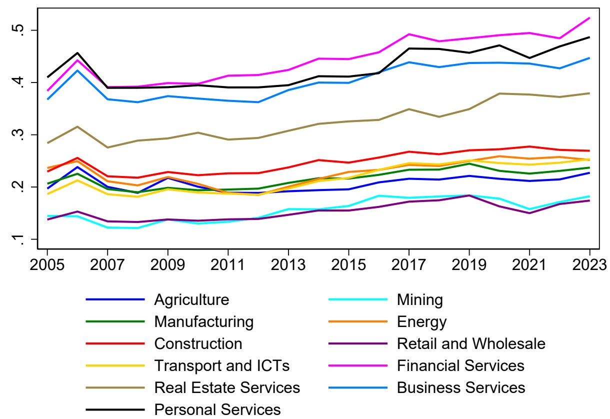
Note. This table reports the median output elasticities for materials, labor, and capital, as well as the implied returns to scale (RTS), across 11 1-digit sectors. Elasticities are estimated using firm-level data and are aggregated by taking the median across 6-digit industries within each sector. RTS is computed as the sum of the three factor elasticities. These values capture sector-specific technology parameters used to compute input wedges and firm level productivities.

Figure 15: Median factor revenue shares evolution by sector

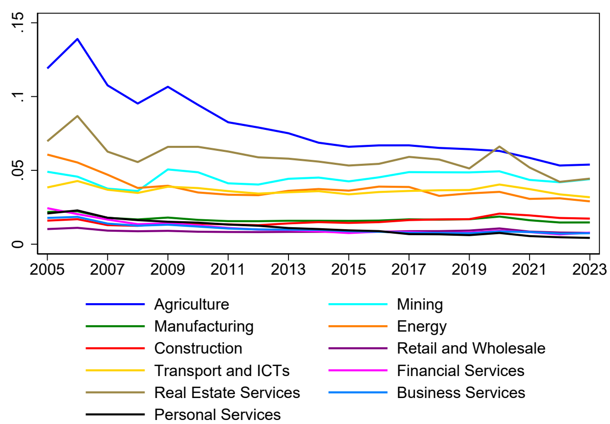
Panel A: Materials share



Panel B: Labor share



Panel C: Capital share



Note. This figure shows the evolution of median firm-level revenue shares by input factor—materials (Panel A), labor (Panel B), and capital (Panel C)—across 1-digit aggregate sectors over time. Revenue shares are computed as the ratio of input expenditures to firm revenue and reflect observed cost structure rather than technology.